FINITE-RANGE SEPARABLE PAIRING INTERACTION WITHIN NEW N^{3}LO DFT APPROACH

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Abstract. For over four decades, the Skyrme functional within various parametrizations has been used to calculate nuclear properties. In the last few years there was a number of attempts to improve its performance and introduce generalized forms. In particular, the most general phenomenological quasi-local energy density functional, which contains all combinations of density, spin-density, and their derivatives up to the sixth order (N^{3}LO), was proposed in reference [1]. Since in the phenomenological functional approaches the particle-particle (pp) interaction channel is treated independently from the particle-hole (ph) channel, there remains a question of what pairing interaction is suitable to use within the N^{3}LO energy functional. In our study, we use the separable, finite-range, translationally invariant form given in [2], which we generalize to the arbitrary angular momentum channel. We discuss the application of this pairing interaction within the N^{3}LO energy functional.

Keywords: separable pairing interaction, N3LO energy density functional
PACS: 21.60.Jz

CONSTRUCTION OF N^{3}LO FUNCTIONAL

The most general phenomenological quasi-local energy density functional, which contains all combinations of density, spin-density, and their derivatives up to the sixth order N^{3}LO - see [1] - has the form:

$$\mathscr{H}(\vec{r}) = \sum_{mI,nLvJ} \sum_{n'L'v'J'} C^{n'L'v'J'}_{mI,nLvJ} \left[\rho_{n'L'v'J'}(\vec{r})[D_{mI}\rho_{nLvJ}(\vec{r})]_{J'}\right]_0 \ . \tag{1}$$

Each term in the summation in Eq. (1) can be characterized by its order, which is defined as:

$$\text{order} = m + n + n' \ . \tag{2}$$

This general EDF contains a large number of coupling constants $C^{n'L'v'J'}_{mI,nLvJ}$; however, there are several symmetries (spherical, Galilean, gauge) that significantly reduce the number of terms in the energy density (1) (see [1] for details).

FINITE-RANGE SEPARABLE PAIRING INTERACTION

In the phenomenological-functional approaches, the particle-particle (pp) channel is treated independently of the particle-hole (ph) channel. Traditionally, for the Skyrme functional, the delta pairing interaction have been used. The main disadvantage of this pairing interaction is, however, its cut-off dependence, as discussed, e.g., in [3]. This requires using renormalization or regularization procedures (see [4] and references cited therein), which may depend on the order of derivatives appearing in the mean-field Hamiltonian. Because of this reason, we decided to apply in the pp channel of our phenomenological functional the finite-range separable pairing interaction, as it was introduced in [2], which does not lead to any cut-off dependence.

The detailed derivation of the finite-range separable pairing interaction is presented elsewhere - see in [5]. Let us only summarize the final form of the pairing energy:

$$\begin{aligned}E^{pp} = &-\frac{1}{4}G_{+}\sum_{J}\sum_{N}\sum_{\mu\nu\mu'\nu'} V^{J,NJ}_{\mu\nu}V^{J,NJ}_{\mu'\nu'}\Big(\langle\Psi_{\alpha_\mu j_\mu}||\kappa_0'^{+J}||\Psi_{\alpha_\nu j_\nu}\rangle\langle\Psi_{\alpha'_\mu j'_\mu}||\kappa_0^{J}||\Psi_{\alpha'_\nu j'_\nu}\rangle + \\ &+\langle\Psi_{\alpha_\mu j_\mu}||\kappa_1'^{+J}||\Psi_{\alpha_\nu j_\nu}\rangle\langle\Psi_{\alpha'_\mu j'_\mu}||\kappa_1^{J}||\Psi_{\alpha'_\nu j'_\nu}\rangle\Big) \\ &-\frac{1}{4}G_{-}\sum_{J}\sum_{N}\sum_{\mu\nu\mu'\nu'} V^{J,NJ}_{\mu\nu}V^{J,NJ}_{\mu'\nu'}\Big(\langle\Psi_{\alpha_\mu j_\mu}||\kappa_0'^{+J}||\Psi_{\alpha_\nu j_\nu}\rangle\langle\Psi_{\alpha'_\mu j'_\mu}||\kappa_1^{J}||\Psi_{\alpha'_\nu j'_\nu}\rangle + \\ &+\langle\Psi_{\alpha_\mu j_\mu}||\kappa_1'^{+J}||\Psi_{\alpha_\nu j_\nu}\rangle\langle\Psi_{\alpha'_\mu j'_\mu}||\kappa_0^{J}||\Psi_{\alpha'_\nu j'_\nu}\rangle\Big). \qquad (3)\end{aligned}$$

In the rel. (3) we used the coupling constants $G_\pm = (G_n \pm G_p)/2$ and the separable interaction matrix elements $V^{J,NJ}_{\mu\nu}$:

$$V^{J,NL}_{\mu\nu} = \sqrt{(2j_\mu+1)(2j_\nu+1)}\begin{Bmatrix} l_\mu & l_\nu & J \\ \frac{1}{2} & \frac{1}{2} & 0 \\ j_\mu & j_\nu & J\end{Bmatrix}\sum_{nl} M^{NLnl}_{n_\mu l_\mu n_\nu l_\nu}\sqrt{4\pi}\left(\int_0^\infty r^2 \mathrm{d}r P(r) R_{nl}(r,b_r)\right), \qquad (4)$$

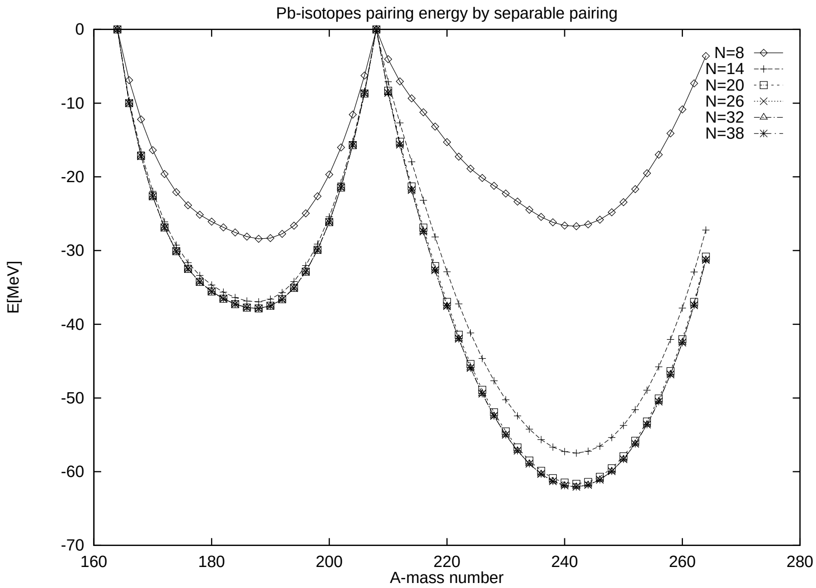
which include the Talmi-Moshinsky brackets $M^{NLnl}_{n_\mu l_\mu n_\nu l_\nu}$, a 9j-symbol coming from the coupling of the two nucleons to total angular momentum J, and a radial integral which can be calculated analytically:

$$\int_0^\infty r^2 \mathrm{d}r P(r) R_{nl}(r,b_r) = \frac{1}{b_r^{3/2}}\sqrt{\frac{(n+\frac{1}{2})\,!}{2n\,!}}\,\frac{1}{m}\sum_{i=1}^{m}\frac{1}{(4\pi a_i^2)^{3/2}}\left(\frac{4a_i^2 b_r^2}{1+2a_i^2 b_r^2}\right)^{3/2}\left(\frac{1-2a_i^2 b_r^2}{1+2a_i^2 b_r^2}\right)^{n}\delta_{l0}\,. \qquad (5)$$

The coupling constants G_+ and G_-, and m constants a_i are the free parameters of the model and their values will be fitted.

CALCULATIONS AND PRELIMINARY RESULTS

Within the aim to make a qualitative test of our method, we calculated the pairing energies of the chain of lead isotopes ($A = 164 - 264$). The energies were calculated by the Hartree-Fock-Bogolyubov method with the finite-range separable pairing force



acting in the pp channel and the Skyrme SLy4 interaction acting in the ph channel. It can be seen that the pairing energies rapidly converge already for $N = 20$ HO shells.

We used the following set for $m = 1$:

$$G_{+} = 738\,\mathrm{MeV\,fm}^3, \quad G_{-} = 0, \quad a_1 = 0.636\,\mathrm{fm}.$$

These values were fixed [2] to reproduce the gap equation of nuclear matter calculated by the Gogny force.

Further refinement of the nuclear pairing properties can be obtained by adding more Gaussian terms, and hence by fitting more parameters in the phenomenological pairing force, i.e. with $m \geq 2$. This will be object of our future effort.

ACKNOWLEDGMENTS

This work is supported by the Academy of Finland and the University of Jyväskylä within the FIDIPRO programme and by the Polish Ministry of Science and Higher Education under Contract No. N N 202 328234.

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