Proc. 12th Int. Conf. on Hypernuclear and Strange Particle Physics (HYP2015)
JPS Conf. Proc. **17**, 012006 (2017)
https://doi.org/10.7566/JPSCP.17.012006

Mean Field and Beyond Mean Field Calculations of Λ Hypernuclei

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(Received December 21, 2015)

We introduce a novel method to calculate spectra of single Λ hypernuclei within mean-field and beyond mean-field approach by using realistic NN and ΛN interactions. We present first implementation of this method - so far at the level of mean field - to describe spectra of three hypernuclei - ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O and ${}^{21}_{\Lambda}$Ne using the chiral NNLO$_{opt}$ and YNG ESC08c forces.

KEYWORDS: Mean Field, Hartree-Fock Method, Hypernuclear Spectrum

1. Introduction

Self-consistent mean-field models are well established for describing nuclear structure - see [1] and citations therein. These models are usually used within three variants - Skyrme energy density functionals, Gogny force and relativistic mean-field Lagrangian. All these variants are phenomenological from the point of view that they start from purely effective interactions with no connection to realistic NN forces which reproduce NN scattering data. There are only few works which try to build the nuclear mean-field directly from realistic NN forces [2, 3].

Mean-field calculations of hypernuclei have been performed up to now solely using phenomenological approaches, such as the Skyrme Hartree Fock model [5–7] or relativistic mean field (RMF) model [8, 9]. The density dependent RMF approach based on realistic YN interactions was used for the description of hypernuclear matter [10]. On the other hand, s-shell hypernuclei have been studied within Faddeev or Yakubovsky equations based on realistic YN and NN interactions including $\Lambda - \Sigma$ conversion [11]. Recently, first ab-initio calculations of light hypernuclei have been performed within No-Core Shell Model [12], Antisymmetrized Molecular Dynamics [16], and Fermionic Molecular Dynamics [13]. It is thus timely to employ mean-field models based on realistic NN and YN interactions in calculations of hypernuclear structure.

The paper is organized as follows. After short introduction we briefly describe the formalism of our model in section 2. In section 3 we present details of our calculations, introduce applied NN and ΛN interactions and discuss the spectra of three hypernuclei - ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O and ${}^{21}_{\Lambda}$Ne - calculated at the level of mean field. In section 4 we discuss further extensions of the calculations.

2. Methodology

We calculate spectra of single Λ hypernuclei by using Hamiltonian of the form

$$H = T + V^{NN} + V^{\Lambda N} - T_{CM}, \tag{1}$$

where $T = \sum_a p_a^2/2m$ is kinetic term, V^{NN} is a realistic NN interaction, $V^{\Lambda N}$ is a realistic ΛN interaction and $T_{CM} = (\sum_a p_a^2 + \sum_{a\neq b} \vec{p}_a.\vec{p}_b)/2mA$ is a centre of mass term. If we introduce the creation

(annihilation) operators $a_i^\dagger$ (a_i) for nucleons and $c_i^\dagger$ (c_i) for Λ, we can express the Hamiltonian (1) in the formalism of second quantization

$$H = \sum_i t_{ij}^N a_i^\dagger a_j + \sum_i t_{ij}^\Lambda c_i^\dagger c_j + \frac{1}{4}\sum_{ijkl} V_{ijkl}^{NN} a_i^\dagger a_j^\dagger a_l a_k + \sum_{ijkl} V_{ijkl}^{N\Lambda} a_i^\dagger c_j^\dagger c_l a_k, \tag{2}$$

with the kinetic matrix elements $t_{ij} = (1 - 1/A) < i|p^2/2m|j >$, antisymmetrized NN interaction matrix elements $V_{ijkl}^{NN} =< ij|(V^{NN} - \vec{p}_1.\vec{p}_2/2mA)|kl - lk >$ and ΛN interaction matrix elements $V_{ijkl}^{N\Lambda} =< ij|(V^{\Lambda N} - \vec{p}_1.\vec{p}_2/2mA)|kl >$, all expressed in the harmonic oscillator (HO) basis.

First we solve the nuclear part of Hamiltonian (2) ($\sum_i t_{ij}^N a_i^\dagger a_i + \frac{1}{4}\sum_{ijkl} V_{ijkl}^{NN} a_i^\dagger a_j^\dagger a_l a_k$) within the Hartree-Fock approximation. As a result we obtain the wave function $| HF >$ as well as the nucleon densities $\rho_{lk}^N =< HF\ |a_k^\dagger a_l|\ HF >$ of the nucleon core of a hypernucleus. Then we calculate the single-particle energies e_i^Λ by diagonalizing the matrix $(t_{ij}^\Lambda + U_{ij}^{N\Lambda})$ where

$$U_{ij}^{N\Lambda} = \sum_{\tau=p,n}\sum_{kl} V_{kilj}^{N\Lambda}\rho_{lk}^N, \tag{3}$$

assuming that the Λ hyperon interacts with the mean field of the core nucleus. The hypernuclear wave function at the level of mean-field approximation is $|i >= c_i^\dagger|HF >$.

Primary goal is to develop a scheme for the description of hypernuclei which will allow us to include the beyond mean-field correlations and core polarization effects which have not been taken into account in this paper. A possible way to achieve that is to couple Λ single-particle states with one-phonon or possibly multi-phonon excitations of the core nucleus by using an Equation of Motion Phonon Model (EMPM) [4] which treats nuclear excitations in multiphonon basis. In this method the Tamm-Dancoff phonon operators $Q_\nu^\dagger$ are used to build Hilbert space spanned by one-, two- and three-phonon configurations. Here, for simplicity, we show how to treat the coupling of the Λ particle to one-phonon states. We assume Hilbert space spanned by hypernuclear states $|\Gamma >\in \{c_i^\dagger|HF >, c_j^\dagger Q_\nu^\dagger|HF >\}$. Then we introduce general superpositions of the basis states $|\Gamma >$:

$$|\alpha >= \sum_\Gamma C_\Gamma^\alpha|\Gamma >, \tag{4}$$

and solve the eigenvalue problem

$$\sum_{\alpha'} < \alpha|H|\alpha' > C_\Gamma^{\alpha'} = E_\alpha C_\Gamma^\alpha. \tag{5}$$

3. Results

We present proof-of-principle calculations for three different hypernuclei - ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O and ${}^{21}_{\Lambda}$Ne. The calculations were done in the harmonic oscillator (HO) basis with $\hbar\omega$ = 16 MeV. The configuration space includes states up to the major oscillator shell N_{max}=10.

We used the realistic NN interaction NNLO$_{opt}$ [14]. It is a chiral next-to-next-to leading order potential with parameters optimized to minimize the effect of three-body interactions, which makes this force useful for many-body calculations (for more details about the optimization procedure see [14]). The calculations of hypernuclei were performed with the YNG interaction derived from the Nijmegen model ESC08 [15] within its latest version called ESC08c [16]. It is represented in a three-range Gaussian form:

$$G(r; k_F) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\ \exp(-r^2/\beta_i^2). \tag{6}$$

For more details including the values of the parameters a_i, b_i, c_i, β_i see [16]. The explicit dependence on Fermi momentum k_F reflects the density dependence of G matrix. The symmetric (SLS) and the antisymmetric spin-orbit (ALS) terms are included within the Scheerbaum approximation [17]. It should be noted that the ΛN-ΣN coupling term in ESC08c is renormalized into the ΛN-ΛN part of G-matrix interaction, giving rise to an important part of the density dependence [16].

The Fermi momentum k_F in eq. (6) can be either treated as an independent parameter of the model or can be related the average density $< \rho >$ of a hypernucleus within the Thomas-Fermi approximation, $k_F = \left(3/2\pi^2 < \rho >\right)^{1/3}$. Calculations reported here were performed for $k_F = 1.26$ fm^{-1} which gives Λ s.p. energies close to experimental values. In Figure 1 we present the lowest Λ s.p. energies e_i^Λ in ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O and ${}^{21}_{\Lambda}$Ne. We show also the available experimental values for comparison [18].

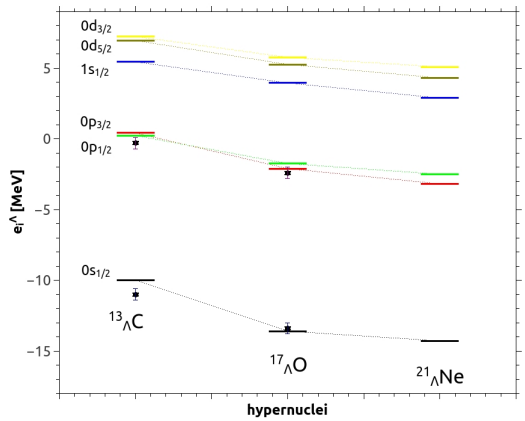

Fig. 1. The lowest Λ s.p. levels in ${}^{13}_{\Lambda}$C, ${}^{17}_{\Lambda}$O and ${}^{21}_{\Lambda}$Ne calculated in HO basis with $\hbar\omega = 16$ MeV and N_{max}=10. The Fermi momentum $k_F = 1.26$ fm^{-1} was used. The experimental data [18] are shown as stars with error bars.

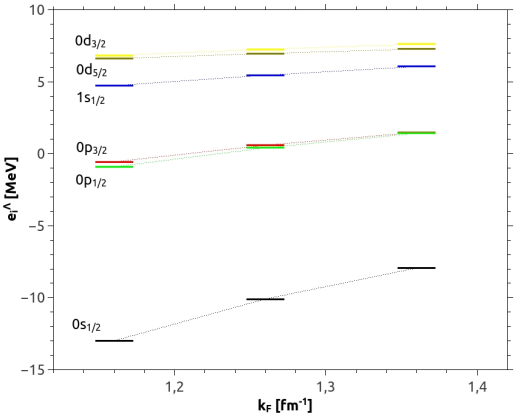

Fig. 2. The Λ s.p. energies in ${}^{13}_{\Lambda}$C calculated with the momenta $k_F = 1.16$ fm^{-1}, $k_F = 1.26$ fm^{-1} and $k_F = 1.36$ fm^{-1}.

In Figure 2 we demonstrate how the low lying Λ s.p. spectrum in ${}^{13}_{\Lambda}$C depends on the Fermi momentum k_F. The Λ hyperon is less bound for higher value of k_F and the relative distances between Λ s.p. levels decrease. We anticipate that after including the Λ particle phonon coupling into our calculations as defined in (4,5), the Λ s.p. energies will be systematically shifted down. Such effect of particle phonon coupling generally occurs in nuclear structure calculations [19]. Therefore, a larger value of k_F will be necessary to get reasonable Λ s.p. spectra.

Important issue in hypernuclear structure studies is the spin-orbit splitting. In our calculations, the order of the $0p_3/2$ and $0p_{1/2}$ Λ s.p. levels is reversed, i.e. $e(0p_{3/2}) > e(0p_{1/2})$, in carbon. This is a consequence of the too strong ALS term which weakens the effect of the SLS term. This is illustrated in Table I where we present the Λ spin-orbit splitting for the cases when we multiply the ALS term by a scaling factor α.

4. Perspectives

There are several points in our calculations which are left for further improvements. So far we omitted three-body NNN and ΛNN forces. The effect of NNN force is minimized by the optimization procedure of NNLO$_{opt}$ which, however, does not mean that this effect is fully negligible [14]. We intend to mimic the effect of three-body interactions by adding a density-dependent two-body term in

Table I. The spin-orbit splitting energy $e(0p_{1/2})$-$e(0p_{3/2})$ [MeV] calculated with (SLS + $\alpha \times$ ALS) terms for different values of α.

α	1.0	0.5	0.0
${}^{13}_{\Lambda}$C	-0.18	0.22	0.62
${}^{17}_{\Lambda}$O	0.36	0.79	1.22
${}^{21}_{\Lambda}$Ne	0.68	1.37	2.16

the same manner as in ref. [3]. In longer perspective we intend to implement directly the chiral NNN forces into our Hamiltonian. It will be also desirable to include a suitable ΛNN interaction.

We are going to extend our calculations beyond the mean-field approach. In the first step we include the Λ particle phonon coupling by solving equations (4,5). Another extension is the treatment of single Λ hypernuclei with the even-odd nucleon core. These calculations are in progress.

5. Acknowledgments

This work was partly supported by the GACR grant P203/15/04301S. Highly appreciated was the access to computing and storage facilities provided by the MetaCentrum under the program LM2010005 and the CERIT-SC under the program Centre CERIT Scientific Cloud, part of the Operational Program Research and Development for Innovations, Reg. no. CZ.1.05/3.2.00/08.0144. P. V. thanks RIKEN for the kind hospitality during his stay.

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