

PAPER • OPEN ACCESS

## Odd nuclei spectroscopy within a self-consistent multiphonon approach

To cite this article: G. De Gregorio *et al* 2018 *J. Phys.: Conf. Ser.* **966** 012007

View the [article online](#) for updates and enhancements.

# Odd nuclei spectroscopy within a self-consistent multiphonon approach

G. De Gregorio<sup>1,2,3</sup>, F. Knapp<sup>4</sup>, N. Lo Iudice<sup>2,3</sup>, P. Veselý<sup>1</sup>

<sup>1</sup>Nuclear Physics Institute, Czech Academy of Sciences, 250 68 Řež, Czech Republic

<sup>2</sup>Dipartimento di Fisica, Università di Napoli Federico II, Naples, Italy

<sup>3</sup>INFN Sezione di Napoli, Naples, Italy

<sup>4</sup>Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

E-mail: [degregorio@na.infn.it](mailto:degregorio@na.infn.it)

**Abstract.** The extension to odd nuclei of the equation of motion phonon method (EMPM) is briefly outlined. It derives and solves iteratively a set of equations of motion which yield an orthonormal basis of states composed of an odd particle coupled to orthonormal multiphonon states, also generated within the EMPM, describing the excitations of a doubly magic core. The basis so obtained is used to solve the eigenvalue problem in the full particle-phonon space. We have applied the method to the odd neighbors of  $^{16}\text{O}$  using an optimized chiral potential in a space spanned by particle-core basis states which include up to three-phonons. The calculations have shown that the multiphonon states enhance enormously the density of levels and compress the whole spectrum, consistently with the data. They also contribute substantially to improve the agreement with the experimental moments and transitions strengths.

## 1. Introduction

The correction induced by the core excitations on the single particle energies of odd nuclei is a longstanding problem, and, to a large extent, not yet solved. One of the first attempts was made during the '60s, within shell model, by including 2p-1h and 4p-3h configurations [1]. The necessity of complex configurations for describing the odd nuclei spectra was also stressed by a weak coupling model [2].

Of different nature is the particle-vibration coupling (PVC) model [3, 4]. It couples an odd particle to the collective excitations of the core commonly described in random-phase approximation (RPA). Recently, PVC calculations were performed within the framework of energy density functionals deduced from Skyrme forces [5, 6] or relativistic meson-nucleon Lagrangians [7, 8] or from the theory of finite Fermi systems [9].

Calculations using chiral forces were performed within the context of self-consistent Green's function theory [10, 11], no-core shell model (NCSM) [12] and coupled cluster theory [13, 14, 15].

Our proposal is the Equation of Motion Phonon Method (EMPM) [16, 17, 18]. It has been applied mainly to study the dipole response in neutron rich nuclei [19, 20]. Recently it was formulated in a Hartree-Fock-Bogolyubov basis and adopted to study the full spectrum as well as the dipole response of neutron-rich  $^{20}\text{O}$  [21].

In its extension to odd nuclei [22, 23], it derives and solves iteratively a set of equations of motion to generate an orthonormal basis of states composed of a valence particle coupled to



$n$ -phonon states ( $n = 1, 2, \dots$ ), also generated within the EMPM, describing the multiphonon excitations of a doubly magic core. The basis is then adopted to solve the full eigenvalue problem. The formalism does not rely on approximations, and takes into fully account the Pauli principle.

## 2. Outline of the Equation of Motion Phonon Method for odd nuclei

Let us consider the Hamiltonian

$$H = H_0 + V. \quad (1)$$

$H_0$  is the one-body term

$$H_0 = \sum_r [r]^{1/2} \epsilon_r \left( a_r^\dagger \times b_r \right)^0, \quad (2)$$

where  $a_r^\dagger = a_{x_r j_r m_r}^\dagger$  ( $b_r = (-)^{j_r + m_r} a_{x_r j_r m_r}$ ) creates (annihilates) a particle of energy  $\epsilon$ ,  $[r]$  stands for  $[r] = 2j_r + 1$  and the symbol  $\times$  denotes angular momentum coupling.  $V$  is a two-body potential

$$V = \frac{1}{4} \sum_{rsqt\sigma} [\sigma]^{1/2} F_{rsqt}^\sigma \left[ \left( a_r^\dagger \times b_s \right)^\sigma \times \left( a_q^\dagger \times b_t \right)^\sigma \right]^0, \quad (3)$$

where  $F^\sigma$  is derived from the two-body potential  $V^\Omega$  through the Pandya transformation

$$F_{rsqt}^\sigma = \sum_\Omega [\omega] (-)^{(r+t-\sigma-\Omega)} W(rsqt; \sigma\Omega) V_{rstq}^\Omega. \quad (4)$$

Here  $W(rsqt; \sigma\Omega)$  are Racah coefficients.

The main goal of the method is to derive for an odd system a basis of correlated particle-core states of the form

$$| \nu_n \rangle = \sum_{p\alpha_n} C_{p\alpha_n}^{\nu_n} | (p \times \alpha_n)^{\nu} \rangle = \sum_{p\alpha_n} C_{p\alpha_n}^{\nu_n} (a_p^\dagger \times | \alpha_n \rangle)^{\nu} \quad (5)$$

from a set of states composed of an odd particle  $p$  coupled to a a basis of correlated  $n$ -phonon core states

$$| \alpha_n \rangle = \sum_{\lambda\alpha_{n-1}} C_{\lambda\alpha_{n-1}}^{\alpha_n} | (\lambda \times \alpha_{n-1})^{\alpha_n} \rangle = \sum_{\lambda\alpha_{n-1}} C_{\lambda\alpha_{n-1}}^{\alpha_n} (O_\lambda^\dagger \times | \alpha_{n-1} \rangle)^{\alpha_n} \quad (6)$$

also derived within the EMPM [18]. In the above equation

$$O_\lambda^\dagger = \sum_{ph} c_{ph}^\lambda (a_p^\dagger \times b_h)^\lambda \quad (7)$$

is the TDA phonon operator, the building block of the multiphonon basis states.

In perfect analogy with the procedure adopted for even nuclei [18] we start with the equations of motion

$$\langle \alpha \| [b_p, H] \| \nu \rangle = (E_\nu - E_\alpha) X_{p\alpha}^{(\nu)}, \quad (8)$$

where  $E_\nu$  are the eigenvalues to be determined and

$$X_{p\alpha}^{(\nu)} = \langle \alpha \| b_p \| \nu \rangle = \sum_{p'\alpha'} \mathcal{D}_{p\alpha p'\alpha'}^\nu C_{p'\alpha'}^\nu. \quad (9)$$

Here

$$\mathcal{D}_{p\alpha p'\alpha'}^\nu = \langle (p \times \alpha)^{\nu} | (p' \times \alpha')^{\nu} \rangle \quad (10)$$

is the overlap matrix which reintroduces the exchange terms among the odd particle and the  $n$ -phonon states and, thereby, re-establishes the Pauli principle. After expanding the commutator in the Eq. (8), we obtain

$$\sum_{p'\alpha'p_1\alpha_1} \left\{ (\epsilon_p + E_\alpha - E_\nu) \delta_{pp'} \delta_{\alpha\alpha'}^{(n)} + \mathcal{V}_{p\alpha p'\alpha'}^{(\nu_n)} \right\} \mathcal{D}_{p'\alpha'p_1\alpha_1}^\nu C_{p_1\alpha_1}^\nu = 0, \quad (11)$$

where

$$\mathcal{V}_{p\alpha p'\alpha'}^{(\nu_n)} = \sum_{\sigma tq} [\sigma]^{1/2} W(\alpha\sigma\nu p'; \alpha'p) \langle \alpha' || [a_t^\dagger \times b_q]^\sigma || \alpha \rangle F_{pp'tq}^\sigma \quad (12)$$

is the particle-phonon potential. Eq. (11) represents an eigenvalue equation in the over-complete basis  $| (p \times \alpha_n)^v \rangle$  within the  $n$ -phonon particle-core subspace. Following the procedure outlined in Refs. [16, 17], based on the Cholesky decomposition method, we extract a basis of linearly independent states  $| (p \times \alpha_n)^v \rangle$  to obtain a non singular eigenvalue equation. Its iterative solution yields the particle-core states  $| \nu_n \rangle$  (5) of energies  $E_{\nu_n}$  for  $n = 1, 2, \dots$ , which, together with the single particle states  $| \nu_0 \rangle$ , form an orthonormal basis.

The eigenvalue problem in the full space spanned by  $\{ | \nu_0 \rangle, | \nu_1 \rangle, \dots | \nu_n \rangle, \dots \}$  is defined by

$$\sum_{\nu_n'} \left\{ (E_{\nu_n} - \mathcal{E}_\nu) \delta_{\nu_n \nu_n'} + \mathcal{V}_{\nu_n \nu_n'}^v \right\} \mathcal{C}_{\nu_n'}^{(\nu)} = 0, \quad (13)$$

where  $\mathcal{V}_{\nu_n \nu_n'}^{(v)}$  are non vanishing for  $n' = n + 1$  and  $n' = n + 2$ . The above eigenvalue equation yields the eigenstates

$$| \Psi_\nu \rangle = \sum_{\nu_n} \mathcal{C}_{\nu_n}^\nu | \nu_n \rangle = \sum_{p\alpha_n} \mathcal{C}_{p\alpha_n}^\nu | (p \times \alpha_n)^v \rangle, \quad (14)$$

where  $\nu_n$  are defined by the Eq. (5). The wavefunctions (14) can be used to compute the transition amplitudes of a multipole operator given by

$$\langle \Psi_{\nu'} || \mathcal{M} || \Psi_\nu \rangle = \sum_{nn'} \mathcal{M}_{nn'}^{(\nu\nu')}(\lambda) = \sum_{nn'} \sum_{\nu_n \nu_n'} \mathcal{C}_{\nu_n}^\nu \mathcal{C}_{\nu_n'}^{\nu'} \langle \nu_n' || \mathcal{M}(\lambda) || \nu_n \rangle. \quad (15)$$

If the initial and/or final states have dominant single particle character we can safely use the truncated formula

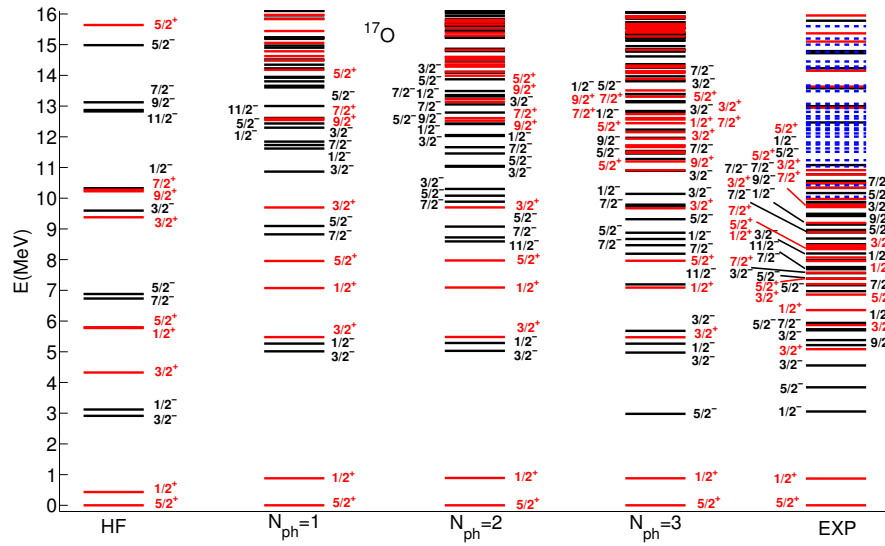
$$\langle \Psi_{\nu'} || \mathcal{M} || \Psi_\nu \rangle \simeq \mathcal{M}_{00}^{(\nu\nu')}(\lambda) + \mathcal{M}_{01}^{(\nu\nu')}(\lambda) + \mathcal{M}_{10}^{(\nu\nu')}(\lambda), \quad (16)$$

where  $\mathcal{M}_{00}^{(\nu\nu')}(\lambda)$ ,  $\mathcal{M}_{01}^{(\nu\nu')}(\lambda)$  and  $\mathcal{M}_{10}^{(\nu\nu')}(\lambda)$  are, respectively, the particle-particle, particle-phonon, and phonon-particle transition amplitudes.

### 3. Calculations and results

We used an Hamiltonian composed of an intrinsic kinetic operator  $T_{int}$  plus the  $NNLO_{opt}$  optimized chiral potential [24] to generate the HF basis in a configuration space including up to the harmonic oscillator major shell  $N_{max} = 15$ . The TDA phonons are derived from a subset of HF states corresponding to  $N = 12$ . Their structure does not change if we use the full HF space. The spurious  $J = 1^-$  TDA components induced by the center of mass (CM) motion have been removed resorting to a Gramm-Schmidt orthogonalization procedure discussed in Ref. [25].

The multiphonon particle-core basis is composed of all one-phonon particle-core states and of a restricted set of two- and three-phonon particle-core states. The inclusion of the three-phonon



**Figure 1.** Theoretical versus experimental [26] spectra of  $^{17}\text{O}$ .  $N_{ph}$  indicates the maximum phonon number. The dashed levels have unknown spin or parity or both.

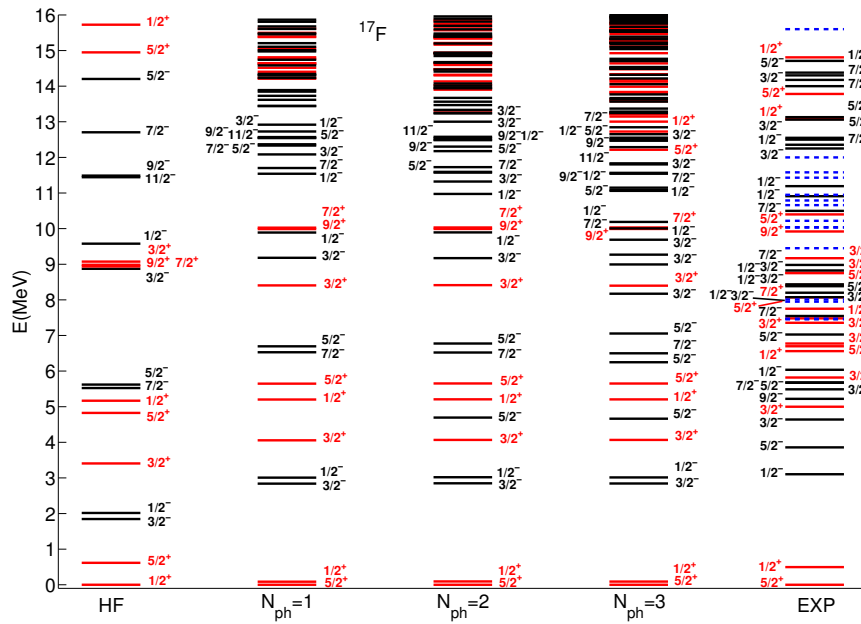
**Table 1.** Ground state electric quadrupole  $Q$  ( $efm^2$ ) and magnetic moments  $\mu$  ( $\mu_n$ ) and  $B(E2; J_i^\pi \rightarrow J_f^\pi)(e^2fm^4)$ . The ground state  $\beta$ -decay  $ft$  value is also included. The experimental data are taken from [26].

		HF	EMPM	EXP
$^{17}\text{O}$	$\mu$	-1.91	-1.83	-1.89
	$Q$	0	0.841	-2.578
	$B(E2; 5/2_1^+ \rightarrow 1/2_1^+)$	0	0.17	2.18(16)
$^{17}\text{F}$	$\mu$	+4.79	+4.63	+4.72
	$Q$	-9.9	-7.6	$\pm 5.8(4)$
	$B(E2; 5/2_1^+ \rightarrow 1/2_1^+)$	40.71	21.89	21.64
	$\log ft$	3.294	3.391	3.358(2)

particle-core states has required some approximations. We ignored the interaction  $\mathcal{V}_{p\alpha_3, p'\alpha'_3}$  in the eigenvalue equation as well as the phonon-phonon potential  $\mathcal{V}_{\lambda\alpha, \lambda'\alpha'}$  determining the core states  $|\alpha_3\rangle$ . We have also neglected the exchange terms between the odd particle and  $|\alpha_3\rangle$  by putting  $\mathcal{D}_{p\alpha_3, p'\alpha'_3} = \delta_{pp'}\delta_{\alpha_3\alpha'_3}$ .

The theoretical spectra obtained for  $^{17}\text{O}$  and  $^{17}\text{F}$  in different multiphonon spaces are compared to one another and with experiments in Fig.1 and 2. The one-phonon states alter appreciably the HF levels. They depress the lowest  $5/2^+$  with respect to the other states thereby enhancing its distance from the other levels. In  $^{17}\text{O}$  this energy shift is beneficial for the low-lying positive parity levels which get closer to the experimental levels of single particle nature. The negative parity states, instead, get pushed at too high energies with the exception of the  $3/2^-$ . In  $^{17}\text{F}$  the energy depression promotes the inversion between the  $1/2^+$  and  $5/2^+$  levels. The correct ground state is thus obtained.

The action exerted by the phonon couplings is similar in both isobars. The inclusion of the two-phonon states enhances the level density only at high energy. The three phonons, by pushing few negative parity states down in energy, enrich the low energy spectra but not sufficiently to reproduce the density of the experimental low-energy level schemes.



**Figure 2.** Theoretical versus experimental [26] spectra of  $^{17}\text{F}$ . The notations are the same as for  $^{17}\text{O}$

The magnetic moments in both  $^{17}\text{O}$  and  $^{17}\text{F}$  are practically determined at the HF level (Tab.1). The weak quenching due to the core brings the total moments slightly more distant from the experimental values. The ground state  $\beta$ -decay of  $^{17}\text{F}$  is also ruled by HF. Indeed, the  $ft$  value comes almost entirely from the transition between the HF components of the  $^{17}\text{O}$  and  $^{17}\text{F}$   $5/2^+$  ground states (Tab.1). The weak quenching caused by the phonon coupling brings the  $ft$  value slightly above the measured quantity.

In  $^{17}\text{O}$ , the calculation underestimates the absolute value of the quadrupole moment by a factor three and the strength of the  $E2$  transition from the ground state  $5/2_1^+$  to  $1/2_1^+$  by an order of magnitude (Tab.1). Since the odd particle is a neutron, and both states have a single particle nature, the contribution to the moments and transition strength comes entirely from the terms  $\mathcal{M}01(E2)$  and  $\mathcal{M}10(E2)$  (Eq. 16) which couple the single particle components of  $5/2_1^+$  and  $1/2_1^+$  to the  $\lambda = 2^+$  particle-phonon pieces of  $1/2_1^+$  and  $5/2_1^+$ , respectively. It seems, therefore, that the amplitudes of the one-phonon components and, in particular, the quadrupole phonon pieces are not sufficiently large.

In  $^{17}\text{F}$ , the quadrupole moment, computed in HF, is  $\sim 1.7$  times the measured value. It gets considerably smaller and closer to experiments once the phonon coupling is included. The coupling is even more effective on the  $5/2_1^+ \rightarrow 1/2_1^+$   $E2$  transition. Once the phonon are included, the  $E2$  strength gets considerably reduced and coincides in practice with the experimental value (Tab.1). This strong effect comes from the indirect coupling among different single particle components of the ground state wave function. This mutual interference causes the quenching of quadrupole moment and transition. In  $^{17}\text{O}$ , this interference has no effect since the odd neutron carries no charge.

#### 4. Conclusions

The above analysis has shown that, while the two phonons have the only effect of enhancing the level density at high energy, the three phonons, through their strong coupling, push down in energy few one-phonon states of negative parity. They enrich thereby the low-energy sector

but not sufficiently to reproduce the experimental level scheme. The phonon contribution to the quadrupole moments and transitions is also substantial but not sufficient to bring those quantities close to the measured data.

This relatively weak phonon action originates from an insufficient particles-phonon and phonon-phonon admixture. In particular, the one-phonon components have too small weights in the states of dominant single particle character, lying generally at low energies, and too large amplitudes in the dominantly particle-core states. This insufficient admixing might be traced back to the large separation between Hartree-Fock levels [25]. A more compressed HF spectrum would yield more closely packed TDA phonons and, therefore, might enhance the coupling of the one-phonon components to the HF vacuum and to the other  $n$ -phonon states. This may be obtained by using a new potential which includes a three-body force.

Only four phonons, through their strong coupling to two-phonon states, would be able to push the positive parity levels down in energy and to enhance the mixing among different  $n$ -phonon components. Including four phonons may be feasible if we resort to approximations analogous to the ones we made here for three phonons.

### Acknowledgements

This work was partly supported by the Czech Science Foundation (P203-13-07117S). Two of the authors (F.K. and P.V.) thank the Istituto Nazionale di Fisica Nucleare (Italy) for financial support. Highly appreciated was the access to computing and storage facilities provided by the Meta Centrum under Program No. LM2010005 and the CERITSC under the program Centre CERIT Scientific Cloud, part of the Operational Program Research and Development for Innovations, Register No. CZ.1.05/3.2.00/08.0144.

### References

- [1] Birkholz J and Beck F 1968 *Phys. Lett. B* **28** 18-21
- [2] Engeland T and Ellis P J 1972 *Nucl. Phys. A* **181** 368-402
- [3] Bohr A and Mottelson B R 1975 *Nuclear Structure* (W. A. Benjamin, New York), Vol II
- [4] Mahaux C, Bortignon P F, Broglia R A, and Dasso C H 1985 *Phys. Rep.* **120**
- [5] Cao L-G, Colò G, Sagawa H, and Bortignon P F 2014 *Phys. Rev. C* **89** 044314
- [6] Co' G, De Donno V, Anguiano M, Bernard R N, and Lallena A M 2015 *Phys. Rev. C* **92** 024314
- [7] Litvinova E and Ring P 2006 *Phys. Rev. C* **73** 044328
- [8] Afanasjev A V and Litvinova E 2015 *Phys. Rev. C* **92** 044317
- [9] Gnezdilov N V, Borzov I N, Saperstein E E and Tolokonnikov S V 2014 *Phys. Rev. C* **89** 034304
- [10] Cipollone A, Barbieri C and Navrátil P 2013 *Phys. Rev. Lett.* **111** 062501
- [11] Somá V, Barbieri C, and Duguet T 2013 *Phys. Rev. C* **87** 011303(R)
- [12] Barrett B R, Navrátil P, and Vary J P 2013 *Prog. Part. Nucl. Phys.* **69** 131
- [13] Hagen G, Papenbrock T, and Hjorth-Jensen M 2010 *Phys. Rev. Lett.* **104** 182501
- [14] Hagen G, Hjorth-Jensen M, Jansen G R, Machleidt R and Papenbrock T 2012 *Phys. Rev. Lett.* **108** 242501
- [15] Jansen G R, Engel J, Hagen G, Navrátil P, and Signoracci A 2014 *Phys. Rev. Lett.* **113** 142502
- [16] Andreozzi F, Knapp F, Lo Iudice N, Porrino A and Kvasil J 2007 *Phys. Rev. C* **75** 044312
- [17] Andreozzi F, Knapp F, Lo Iudice N, Porrino A and Kvasil J 2008 *Phys. Rev. C* **78** 054308
- [18] Bianco D, Knapp F, Lo Iudice N, Andreozzi F and Porrino A 2012 *Phys. Rev. C* **85** 014313
- [19] Knapp F, Lo Iudice N, Veselý P, Andreozzi F, De Gregorio G and Porrino A 2014 *Phys. Rev. C* **90** 014310
- [20] Knapp F, Lo Iudice N, Veselý P, Andreozzi F, De Gregorio G and Porrino 2015, *Phys. Rev. C* **92** 054315
- [21] De Gregorio G, Knapp F, Lo Iudice N and Veselý P 2016 *Phys. Rev. C* **93** 044314
- [22] De Gregorio G, Knapp F, Lo Iudice N and Veselý P 2016 *Phys. Rev. C* **94** 061301(R)
- [23] De Gregorio G, Knapp F, Lo Iudice N and Veselý P 2017 *Phys. Rev. C* **95** 034327
- [24] Ekström A, Baardsen G, Forssén C, Hagen G, Hjorth-Jensen M, Jansen G R, Machleidt R, Nazarewicz W, Papenbrock T, Sarich J, et al. 2013 *Phys. Rev. Lett.* **110** 192502
- [25] Bianco D, Knapp F, Lo Iudice N, Veselý P, Andreozzi F, De Gregorio G, and Porrino A 2014 *J. Phys. G* **41** 025109
- [26] Tilley D, Weller H, and Cheves C 1993 *Nucl. Phys. A* **564** 1