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Radiative corrections for Dalitz decays of π^0, $\eta^{(\prime)}$ and Σ^0

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Montpellier
July 4, 2019

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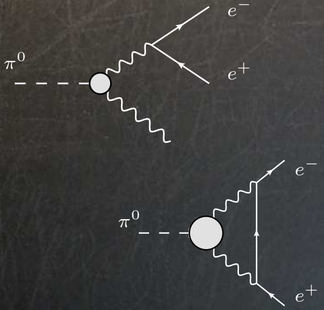
EXCELENCIA SEVERO OCHOA

Rare decay of π^0

Introduction

Decay modes of the neutral pion:

Process	Branching ratio
$\pi^0 \to \gamma\gamma$	$(98.823 \pm 0.034)\,\%$
$\pi^0 \to e^+e^-\gamma$	$(1.174 \pm 0.035)\,\%$
$\pi^0 \to e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \to e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



Rare decay $\pi^0 \to e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to *Drell*, NC (1959)
- suppressed in comparison to the decay $\pi^0 \to \gamma\gamma$ by a factor of $2(\alpha m_e/M_\pi)^2$
  - $\hookrightarrow$ one-loop structure + helicity suppression
  - $\hookrightarrow$ may be sensitive to possible effects of new physics

Rare decay of π^0

KTeV measurement

KTeV-E799-II experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))
↪ precise measurements of branching ratio $\pi^0 \to e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma),\ x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\ x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B_{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma),\ x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
- extrapolate full radiative tail beyond $x > 0.95$ (*Bergström*, Z.Ph.C 20 (1983))
- scale the result back by the overall radiative corrections

↪ final result for lowest order (no final state radiation)

$$B_{\text{KTeV}}^{\text{no-rad}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov*, PRD 75 (2007))

$$B_{\text{SM}}^{\text{no-rad}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

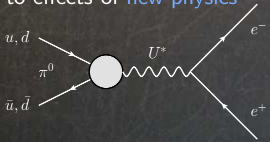
↪ interpreted as 3.3 σ discrepancy between theory and experiment

Rare decay of π^0

New physics?

- very fashionable to ascribe eventual discrepancies to effects of new physics

BUT



- first, look for more conventional solution (i.e. within SM)
  ↪ radiative corrections (usually very important)
  ↪ transition-form-factor modeling: *TH and Leupold*, EPJC 75 (2015)

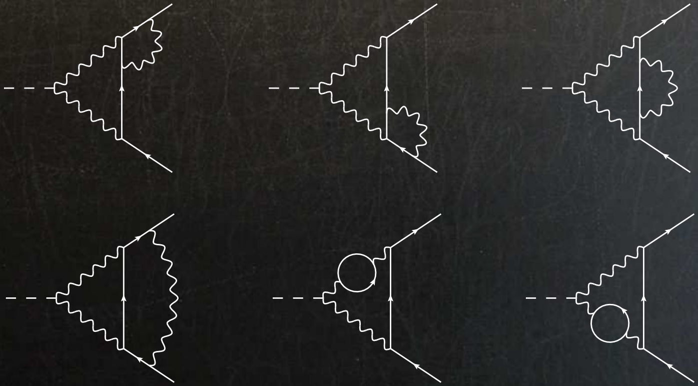
Two-hadron saturation ("LMD+V+P")

$$\mathcal{F}^{\text{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right]$$
$$\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

Radiative corrections for $\pi^0 \to e^+e^-$

Two-loop virtual radiative corrections

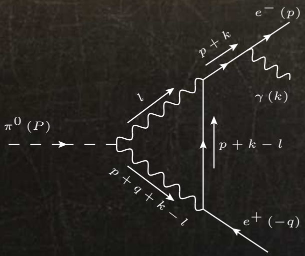
- calculated by *Vaško and Novotný*, JHEP 1110 (2011)

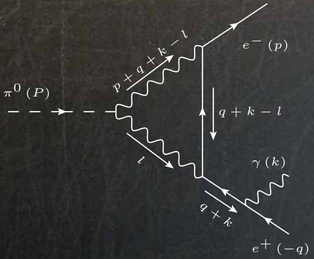


Radiative corrections for $\pi^0 \to e^+e^-$

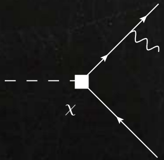
Bremsstrahlung: photon emission from the outer fermion line

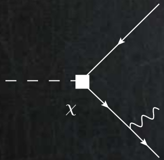
- compensation of IR divergence in 2-loop contributions
  $\hookrightarrow$ *TH, Kampf and Novotný*, EPJC 74 (2014)





- contain UV subdivergences → counter-term tree diagrams with couplig χ





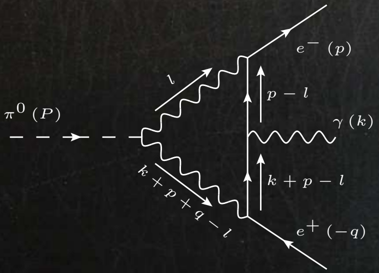
Radiative corrections for $\pi^0 \to e^+e^-$

Bremsstrahlung: photon emission from the inner fermion line (propagator)

Do not forget the third, box diagram, necessary to satisfy the Ward identities

$$\mathcal{M}_{(\lambda)} = \varepsilon^{*\rho}_{(\lambda)}(k)\mathcal{M}^{\text{BS}}_{\rho} \quad \longrightarrow \quad k^\rho \mathcal{M}^{\text{BS}}_{\rho} = 0$$

- finite contribution to bremsstrahlung amplitude



Radiative corrections for $\pi^0 \to e^+e^-$

Final results

Size of the radiative corrections (newly calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$
Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$

- original KTeV vs. SM discrepancy reduced to the $2\,\sigma$ level or less
- contact interaction coupling finite part set to

$$\chi^{(\text{r})}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$$

Dalitz decay of the neutral pion

Introduction

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0 \to e^+e^-(\gamma)\,,\ x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma(\gamma)\,,\ x > 0.2319)}\right|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

↪ Dalitz decay comes into play

- second most important decay channel of the neutral pion
  ↪ branching ratio $(1.174 \pm 0.035)\,\%$
- first studied by *Richard H. Dalitz,* PPSA 64 (1951), whose name it carries
- experimental data of this process provide the information about singly-virtual pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$
  ↪ in particular about its slope parameter a_π

Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Introduction

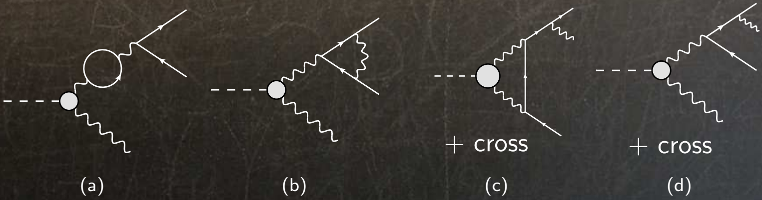
- radiative corrections to the total decay rate of the Dalitz decay
  $\hookrightarrow$ first addressed (numerically) by *Joseph*, NC 16 (1960)

- pioneering study of corrections to the differential decay rate
  $\hookrightarrow$ *Lautrup and Smith*, PRD 3 (1971)
  $\hookrightarrow$ soft-photon approximation

- extended by *Mikaelian and Smith*, PRD 5 (1972)
  $\hookrightarrow$ hard-photon corrections
  $\hookrightarrow$ whole range of bremsstrahlung photon energy
  $\hookrightarrow$ table of values

Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Novel approach



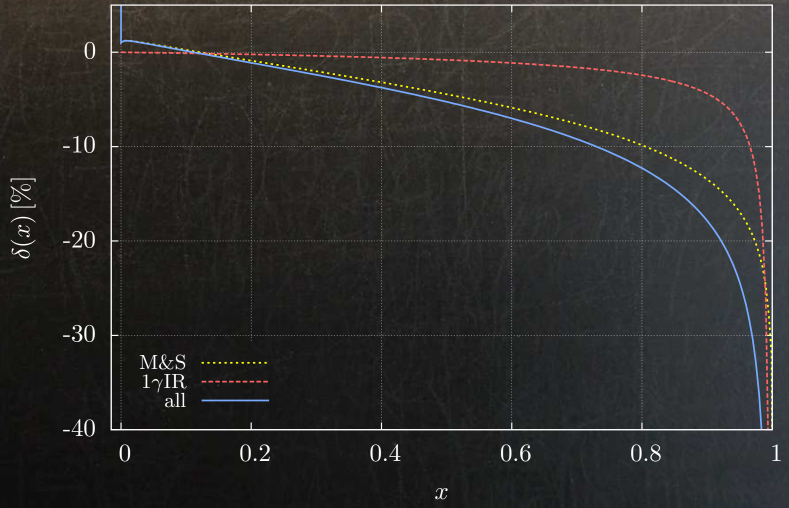
- new calculations motivated by needs of NA48/NA62 experiments at CERN
  $\hookrightarrow$ measure the slope of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$: *Lazzeroni et al.*, PLB 768 (2017)

$$a_\pi^{\text{NA62}} = 3.68(57)\,\%$$

- unlike before no approximation was used
  $\hookrightarrow$ can be used also for related decays $\eta \to \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
  $\hookrightarrow$ propagated into MC generator of NA62 experiment
- *TH, Kampf and Novotný*, PRD 92 (2015)

Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Results



Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Determination of ratio R

Precise and reliable determination of $R \equiv \frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$

↪ for small slope and up to NLO radiative corrections

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x,y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x}\right] \mathrm{d}x\,\mathrm{d}y$$

Conservative estimate for uncertainty (a_π, NNLO): $R = 1.1978(5)(3)\,\%$

↪ chosen $a_\pi^{\text{univ}} = 3.55(70)\,\%$, covers

$$a_\pi^{\text{VMD}} = 3.0\,\%\,, \quad a_\pi^{\text{PDG}} = 3.35(31)\,\%\,, \quad a_\pi^{\text{NA62}} = 3.68(57)\,\%$$

Constraint: $1 \simeq \mathcal{B}(\pi^0 \to \gamma\gamma) + \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \to e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \to \gamma\gamma) = 98.8131(6)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) = 1.1836(6)\,\%$$

TH, Goudzovski and Kampf, PRL 122 (2018)

PDG

$$R = 1.188(35)\,\%\,, \quad \mathcal{B}(\pi^0 \to \gamma\gamma) = 98.823(34)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma) = 1.174(35)\,\%$$

Determination of ratio R

Why it works?

TFF normalization $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)$ drops out in R
↪ TFF dependence solely represented by its shape

for π^0, transfered momentum significantly (kinematically) limited
↪ linear expansion very good approximation
↪ slope a_π constitutes the only parameter of the low-energy QCD sector

allowing for 20 % uncertainty on a_π^{univ} due to:

- smallness of the slope
- strong suppression of the region $x \simeq 1$ where the $a_\pi x$ term matters

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x,y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x}\right] \mathrm{d}x\,\mathrm{d}y$$

‘Recent’ KTeV measurement

Abouzaid et al., arXiv:1902.01375
↪ based on 1999 data and *E. Abouzaid*, Ph.D. thesis (2007)

$$\frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = (1.1559 \pm 0.0046 \pm 0.0106)\,\%$$

TH, Goudzovski and Kampf, PRL 122 (2018)
Conservative estimate for uncertainty (a_π, NNLO): $R = 1.1978(5)(3)\,\%$
↪ chosen $a_\pi^{\text{univ}} = 3.55(70)\,\%$

$\implies$ $3.6\,\sigma$ discrepancy between theory and experiment

PDG average: $R = 1.188(35)\,\%$
↪ most recent (archived ALEPH data) *Beddall and Beddall*, EPJC 54 (2008)

Need for new measurements: e.g. NA62

Dalitz decays of $\eta^{(\prime)}$

Introduction

$\eta^{(\prime)}$ Dalitz decays

- small branching ratios
  $\hookrightarrow$ hadronic decay modes are open

- access to electromagnetic transition form factors
  $\hookrightarrow$ $\eta^{(\prime)}$-meson structure
  $\hookrightarrow$ valuable input for other quantities and e.g. $g-2$ of a muon
  $\hookrightarrow$ radiative corrections crucial to extract relevant information from data

naive rad. corrections for $\eta \to e^+e^-\gamma$: *Mikaelian and Smith*, PRD 5 2890 (1972)

- numerical values correspond to simple change $M_{\pi^0} \to M_\eta$
  $\hookrightarrow$ π^0 case: *Mikaelian and Smith*, PRD 5 1763 (1972)

inclusive radiative corrections
$\hookrightarrow$ no momentum or angular cuts on the bremsstrahlung photon applied

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

What should be taken into account?

The $\eta^{(\prime)}$ case compared to π^0

- larger rest mass

  $\hookrightarrow$ M_η above muon-pair threshold: $M_\eta > 2m_\mu$
  $\hookrightarrow$ $M_{\eta'}$ above lowest-lying resonances: $M_{\eta'} > M_\rho, M_\omega$
    $\hookrightarrow$ sensitive to the widths of resonances
    $\hookrightarrow$ ω narrow, ρ broad resonance in $\pi\pi$ scattering

- strange-flavor content

  $\hookrightarrow$ quark-flavor basis
  Feldmann et al., PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2}\left[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d\right], \quad j^{\mathsf{s}} \equiv \frac{i}{\sqrt{2}}\left[\bar{s}\gamma_5 s\right]$$

- η-η' mixing: $\langle 0|j^A|\eta^B\rangle = B_0 F_\pi f_A \delta^{AB}$, $\langle \eta^A|\eta^B\rangle = \delta^{AB}$, $A, B \in \{\ell, \mathsf{s}\}$

$$|\eta\rangle = \cos\phi\,|\eta^\ell\rangle - \sin\phi\,|\eta^{\mathsf{s}}\rangle$$
$$|\eta'\rangle = \sin\phi\,|\eta^\ell\rangle + \cos\phi\,|\eta^{\mathsf{s}}\rangle$$

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Novel approach

Full set of NLO QED radiative corrections:
TH, Kampf, Leupold and Novotný, PRD 97 (2018)

- compared to previous approach:
  ↪ muon loops + hadronic VP
  ↪ 1γIR at one-loop level
  ↪ form-factor effects (also in BS)
  ↪ higher orders in the final-state-lepton mass not neglected
- general framework: three additional processes
  ↪ also muon decay modes

η case: most of the ingredients in *TH, Kampf and Novotný*, PRD 92 (2015)

η' case: real challenge

↪ resulting framework also applicable to the π^0 case (numerically compatible)
↪ overkill (correction to the correction of order 1 %)

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Virtual corrections

Photon self-energy in the form $\Pi(s) = \Pi_\mathsf{L}(s) + \Pi_\mathsf{H}(s)$

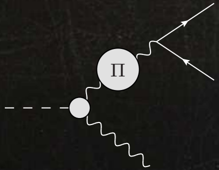
- lepton loops (electrons and as well muons)

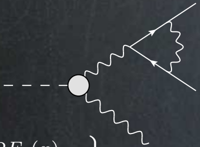
$$\Pi_\mathsf{L}(M_P^2 x) = \frac{\alpha}{\pi} \sum_{\ell'=e,\mu} \left\{ \frac{8}{9} - \frac{\beta_{\ell'}^2}{3} + \left(1 - \frac{\beta_{\ell'}^2}{3}\right) \frac{\beta_{\ell'}}{2} \log\left[-\gamma_{\ell'} + i\epsilon\right] \right\}$$

- hadronic contribution

↪ *Jegerlehner,* Z.Ph.C 32 (1986)

$$\Pi_\mathsf{H}(s) = -\frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_\mathsf{H}(s')\,\mathrm{d}s'}{s - s' + i\epsilon}$$

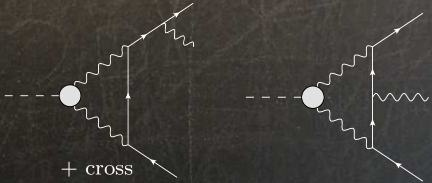




$$\delta^\mathsf{virt}(x,y) = \frac{1}{|1+\Pi(M_P^2 x)|^2} - 1 + 2\,\mathrm{Re}\left\{ F_1(x) + \frac{2F_2(x)}{1+y^2+\frac{\nu^2}{x}} \right\}$$

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

1γIR contribution at one-loop level



1γIR contribution at one-loop level

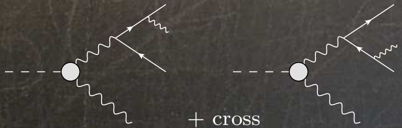
- beyond effective approach
- we don't expect substantial model dependence ↪ VMD-inspired model
  ↪ strange-flavor content and η-η' mixing

$$e^2\mathcal{F}^{\mathsf{VMD}}_{\eta\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_{\mathsf{c}}}{8\pi^2F_\pi}\frac{2e^2}{3}$$

$$\times\left[\frac{5}{3}\frac{\cos\phi}{f_\ell}\frac{M^4_{\omega/\rho}}{(p^2-M^2_{\omega/\rho})(q^2-M^2_{\omega/\rho})} - \frac{\sqrt{2}}{3}\frac{\sin\phi}{f_{\mathsf{s}}}\frac{M^4_\phi}{(p^2-M^2_\phi)(q^2-M^2_\phi)}\right]$$

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Bremsstrahlung



- slope not negligible
- for η: expansion in slope a would be still (somewhat) suitable

$$\mathcal{F}\big((p_\gamma + p_{e^+} + p_{e^-})^2\big) \simeq \mathcal{F}(M_P^2 x)\left[1 + a\,\frac{2p_\gamma \cdot (p_{e^+} + p_{e^-})}{M_P^2}\right]$$

- for η': such an expansion not applicable anymore
  $\hookrightarrow$ BS necessarily depends on the form-factor model

sensitivity to width of ρ meson $\hookrightarrow$ recent dispersive calculations used
Hanhart et al., EPJC 73 (2013), EPJC 77 (2017)

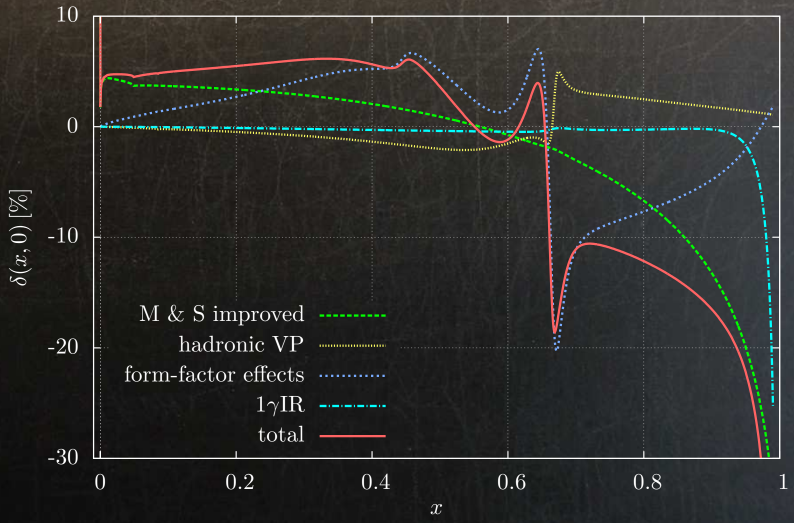
Källén–Lehmann spectral representation → common spectral density function

$$\frac{\mathcal{F}(q^2)}{\mathcal{F}(0)} \simeq 1 + q^2 \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathcal{A}(s)\,\mathrm{d}s}{q^2 - s + i\epsilon}$$

$$\mathcal{A}(s) = w_\omega \mathcal{A}_\omega(s) + w_\phi \mathcal{A}_\phi(s) - \frac{\kappa}{96\pi^2 F_\pi^2}\left[1 - \frac{4m_\pi^2}{s}\right]^{3/2} P(s)\,R(s)\,|\Omega(s)|^2$$

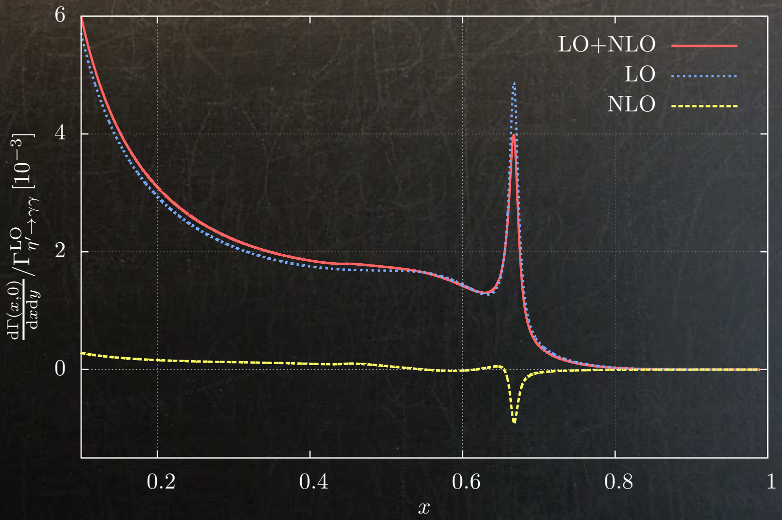
Radiative corrections for $\eta' \to e^+e^-\gamma$ decays

The overall NLO correction $\delta(x,0)$ in comparison to its constituents



Radiative corrections for $\eta' \to e^+e^-\gamma$ decays

The two-fold differential decay width $\mathrm{d}\Gamma(x,0)$ at NLO

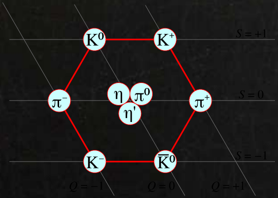


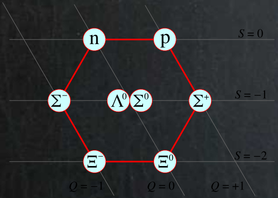
Dalitz decay of Σ^0

Introduction

How are the building blocks distributed inside of the composite objects?
↪ electron-nucleon scattering
  ↪ electromagnetic form factors
  ↪ known LE quantities: el. charge, mag. moment, electric and magnetic radii

Replacement of down quarks (of a nucleon/Δ) by strange quark(s)?
↪ revealing quarks as building blocks of nucleons and hadrons in general
↪ intimate relation among the intrinsic structures of hyperons and nucleons
  ↪ hyperon electromagnetic and transition form factors contain complementary information to the nucleon (and Δ) ones





Dalitz decay of Σ^0

Introduction

Experimental knowledge of hyperons rather limited
$\hookrightarrow$ unstable (electron scattering rather difficult)
$\hookrightarrow$ magnetic moments (and electric charges) known

Form factors at high energies
$\hookrightarrow$ e^+e^- scattering to hyperon + antihyperon
$\hookrightarrow$ direct and transition form factors accessible

At low energies
$\hookrightarrow$ Dalitz decay $Y \to Y'e^+e^-$
$\hookrightarrow$ possibly high statistics in future at FAIR (PANDA: $p\bar{p}$, HADES: pp)

Dalitz decays in baryon-octet sector?

Dalitz decay of Σ^0

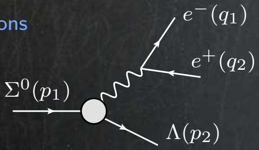
Introduction

Dalitz decay $\Sigma^0 \to \Lambda e^+ e^-$

↪ e^+e^- invariant mass only up to $M_\Sigma - M_\Lambda \simeq 77\,\text{MeV}$

↪ electric and magnetic transition form factors of $\Sigma^0 \to \Lambda$ transition

- ↪ extracting transition radii challenging
- ↪ high-precision measurement required
- ↪ competing with QED radiative corrections



Predictions of electric and magnetic radii

↪ *Granados, Leupold and Perotti*, EPJA 53 (2017)

Dalitz decay of Σ^0

Leading order

$\Sigma^0\Lambda\gamma$ vertex: $\langle 0|j^\mu|\Sigma^0\bar{\Lambda}\rangle = e\bar{v}_\Lambda(\vec{p}_2)\,G^\mu(p_1+p_2)\,u_\Sigma(\vec{p}_1)$, with

$$G^\mu(q) \equiv \left[\gamma^\mu - (M_\Sigma - M_\Lambda)\frac{q^\mu}{q^2}\right]G_1(q^2) - \frac{i\sigma^{\mu\nu}q_\nu}{M_\Sigma + M_\Lambda}G_2(q^2)$$

Define magnetic and electric form factors

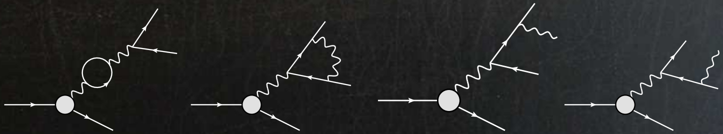
$$G_{\text{M}}(q^2) \equiv G_1(q^2) + G_2(q^2) = \kappa\left(1 + \frac{1}{6}\langle r_{\text{M}}^2\rangle q^2 + \mathcal{O}(q^4)\right)$$

$$G_{\text{E}}(q^2) \equiv G_1(q^2) + \frac{q^2}{(M_\Sigma + M_\Lambda)^2}G_2(q^2) = \frac{1}{6}\langle r_{\text{E}}^2\rangle q^2 + \mathcal{O}(q^4)$$

Matrix element squared dominated by the magnetic part

$$\overline{|\mathcal{M}^{\text{LO}}(x,y)|^2} \simeq 2e^4|G_{\text{M}}(\Delta_M^2 x)|^2\,\frac{(1-x)}{x}\left(1 + y^2 + \frac{\nu^2}{x}\right)$$

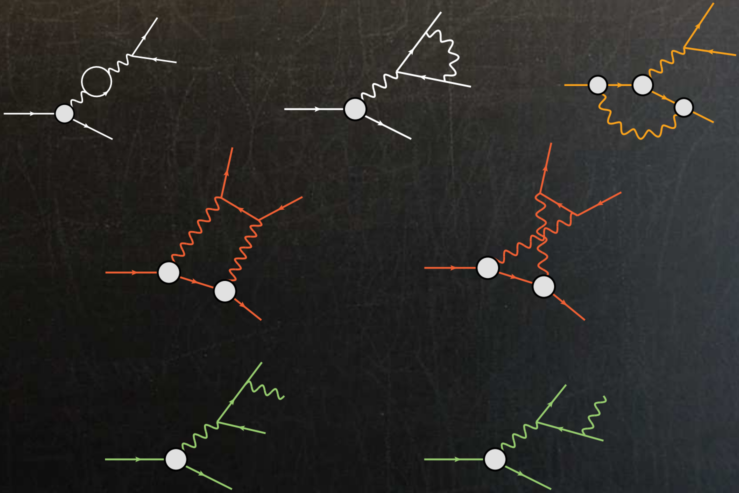
Radiative corrections to the differential decay width in soft-photon approximation
$\hookrightarrow$ *Sidhu and Smith*, PRD 4 3344 (1971)



Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$
Diagrams

TH and Leupold, in preparation

↪ inclusive radiative corrections beyond the soft-photon approximation



Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Bremsstrahlung

Low-energy expansion of the form factors:

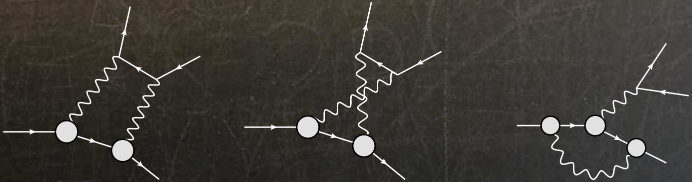
$$G_\mathrm{M}\left((k+q_1+q_2)^2\right) \simeq G_\mathrm{M}\left((q_1+q_2)^2\right)\left\{1+\frac{1}{6}\langle r_\mathrm{M}^2\rangle[2k\cdot(q_1+q_2)]\right\},$$

$$G_\mathrm{E}\left((k+q_1+q_2)^2\right) \simeq G_\mathrm{E}\left((q_1+q_2)^2\right)\left\{1+\frac{2k\cdot(q_1+q_2)}{(q_1+q_2)^2}\right\}.$$

Subsequently, integrate over the energy and emission angle of bremsstr. photon
↪ radiative corrections for inclusive process

Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Virtual corrections



By loop-momenta-power counting, FFs required to regulate the UV region

1γIR: UV-convergence already achieved in the simplest case with constant FFs
$\hookrightarrow$ $G_{\mathsf{E}}(q^2) = G_{\mathsf{E}}(0) = 0$ and $G_{\mathsf{M}}(q^2) = G_{\mathsf{M}}(0) = \kappa$

$$G_1(q^2) = \kappa \frac{q^2}{q^2 - M_V^2}\,, \quad G_2(q^2) = -\kappa \frac{M_V^2}{q^2 - M_V^2}$$

Ansatz satisfying high-energy constraints

$$G_1(q^2) = \kappa \left(3 - \frac{M_V^2 \langle r_{\mathsf{M}}^2 \rangle}{6}\right) \frac{q^2 M_V^4}{(q^2 - M_V^2)^3}\,, \quad G_2(q^2) = -\kappa \frac{M_V^6}{(q^2 - M_V^2)^3}$$

These contributions to the NLO decay width are found to be negligible

Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Results

x \ y	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
0.01	2.50	2.44	2.31	2.18	2.03	1.87	1.65	1.33	0.83	-0.20	-8.26
0.02	2.67	2.61	2.49	2.34	2.19	2.00	1.75	1.40	0.84	-0.26	-5.84
0.03	2.71	2.66	2.55	2.41	2.24	2.04	1.78	1.41	0.83	-0.33	-5.75
0.04	2.71	2.67	2.56	2.42	2.26	2.06	1.79	1.41	0.80	-0.40	-5.84
0.05	2.69	2.65	2.55	2.42	2.26	2.05	1.78	1.39	0.77	-0.47	-5.98
0.06	2.66	2.62	2.53	2.40	2.24	2.04	1.76	1.37	0.73	-0.53	-6.13
0.07	2.61	2.58	2.49	2.37	2.21	2.01	1.74	1.34	0.69	-0.60	-6.29
0.08	2.56	2.53	2.45	2.34	2.18	1.98	1.71	1.30	0.65	-0.66	-6.44
0.09	2.51	2.48	2.41	2.29	2.15	1.95	1.68	1.27	0.60	-0.73	-6.60
0.10	2.45	2.43	2.36	2.25	2.11	1.91	1.64	1.23	0.56	-0.79	-6.75
0.15	2.14	2.12	2.07	1.99	1.87	1.69	1.42	1.01	0.31	-1.12	-7.47
0.20	1.79	1.78	1.75	1.69	1.59	1.43	1.17	0.75	0.04	-1.46	-8.14
0.25	1.43	1.42	1.40	1.36	1.28	1.14	0.89	0.48	-0.26	-1.81	-8.78
0.30	1.05	1.05	1.04	1.01	0.95	0.82	0.59	0.17	-0.57	-2.18	-9.40
0.35	0.65	0.65	0.65	0.64	0.59	0.48	0.26	-0.15	-0.91	-2.57	-10.0
0.40	0.23	0.23	0.24	0.24	0.21	0.11	-0.10	-0.51	-1.28	-2.99	-10.6
0.45	-0.22	-0.22	-0.20	-0.18	-0.20	-0.29	-0.49	-0.89	-1.68	-3.43	-11.3
0.50	-0.71	-0.70	-0.67	-0.64	-0.65	-0.72	-0.91	-1.31	-2.11	-3.91	-11.9
0.55	-1.23	-1.22	-1.19	-1.15	-1.14	-1.20	-1.38	-1.78	-2.59	-4.43	-12.6
0.60	-1.81	-1.79	-1.75	-1.70	-1.68	-1.73	-1.90	-2.30	-3.12	-5.01	-13.3
0.65	-2.46	-2.44	-2.38	-2.32	-2.29	-2.32	-2.49	-2.89	-3.72	-5.65	-14.1
0.70	-3.19	-3.16	-3.11	-3.03	-2.99	-3.01	-3.17	-3.56	-4.41	-6.38	-14.9
0.75	-4.04	-4.01	-3.94	-3.86	-3.80	-3.81	-3.96	-4.36	-5.22	-7.23	-15.9
0.80	-5.06	-5.03	-4.96	-4.86	-4.79	-4.79	-4.93	-5.33	-6.21	-8.26	-17.0
0.85	-6.36	-6.33	-6.24	-6.14	-6.05	-6.04	-6.18	-6.58	-7.47	-9.56	-18.4
0.90	-8.16	-8.12	-8.03	-7.91	-7.81	-7.79	-7.92	-8.32	-9.24	-11.4	-20.3
0.95	-11.2	-11.1	-11.0	-10.9	-10.8	-10.8	-10.9	-11.3	-12.2	-14.4	-23.4
0.99	-18.0	-18.0	-17.9	-17.7	-17.6	-17.6	-17.7	-18.1	-19.0	-21.2	-30.3

Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

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Summary

PRELIMINARY

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Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Integrated decay width

Integrate over the Dalitz plot

$$\frac{\Gamma(\Sigma^0 \to \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \to \Lambda\gamma)} = 5.545 \times 10^{-3}$$

↪ consistent with the NLO result in *Sidhu and Smith*, PRD 4 3344 (1971)

$$\frac{\Gamma(\Sigma^0 \to \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \to \Lambda\gamma)} = (5.532 + 0.627a) \times 10^{-3} \simeq 5.544 \times 10^{-3}$$

LO result:

$$\frac{\Gamma^{\mathsf{LO}}(\Sigma^0 \to \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \to \Lambda\gamma)} = 5.495 \times 10^{-3}$$

$\Longrightarrow$ QED NLO correction to the $\Sigma^0 \to \Lambda e^+ e^-$ decay rate: $\delta = 0.90\,\%$

Summary

All NLO QED radiative corrections for discussed processes are now available
↪ can be taken into account in future experimental analyses

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
  ↪ THS model: *TH and S. Leupold*, EPJC 75 (2015)
- $\pi^0 \to e^+e^-\gamma$
  TH, Kampf and Novotný, PRD 92 (2015)
  ↪ precise determination of R: *TH, Goudzovski and Kampf*, PRL 122 (2018)
- $\eta^{(\prime)} \to \ell^+\ell^-\gamma$
  TH, Kampf, Leupold and Novotný, PRD 97 (2018)

Baryon sector

- $\Sigma^0 \to \Lambda e^+e^-$
  TH, Leupold, in preparation

Ancillary files available together with the papers

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Thank you for listening!