Decays of Neutral Pions

Electromagnetic form factors and radiative corrections

Tomáš Husek[1], Karol Kampf[1], Stefan Leupold[2], Jiří Novotný[1]

[1]Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University in Prague
[2]Institutionen för Fysik och Astronomi, Uppsala Universitet

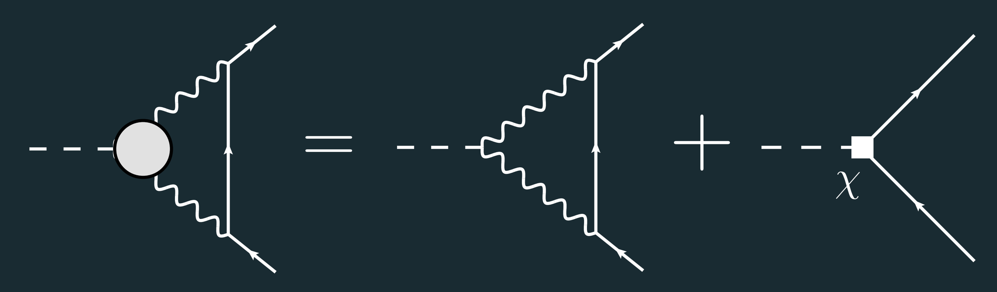
Discrepancy in $\pi^0 \to e^- e^+$ decay

Rare decay $\pi^0 \to e^- e^+$

- precise measurement of branching ratio
  → KTeV experiment at Fermilab
  (*Abouzaid et al.*, PRD (2007) 75)
  $B^{\text{KTeV}}(\pi^0 \to e^- e^+(\gamma),\, x_D > 0.95) = (6.44 \pm 0.33) \times 10^{-8}$
- Standard Model theoretical prediction
  → 3.3 σ disagreement
  (*Dorokhov and Ivanov*, PRD (2007) 75)
- discrepancy not satisfactorily explained yet
- very fashionable to ascribe eventual discrepancies to effects of new physics
- but first look for more conventional solution (i.e. within SM)
  → radiative corrections (usually important)
  → form factor modeling

Leading order

- pions are complicated composite objects
  → elementary interactions are not point-like
- electromagnetic pion transition form factor $F_{\pi^0\gamma^*\gamma^*}$ describes this complexity



LO contribution in QED expansion — its representation as the LO of χPT

- free parameter $\chi^{(r)}(\mu)$ appears in the counter term

$$\chi = [\text{UV-divergent part}] + \chi^{(r)}(\mu)$$

→ unique for every form factor, e.g. $\chi^{(r)}_{\text{KTeV}}(M_\rho) = 6.0 \pm 1.0$

Radiative corrections

Size of the radiative corrections newly calculated

$$\delta(0.95) \equiv \delta^{(2\text{-loop})} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations
  Bergström, Z.Ph.C (1983) 20: $\delta(0.95) = -13.8\,\%$
  Dorokhov et al., EPJC (2008) 55: $\delta(0.95) = -13.3\,\%$
- original KTeV vs. SM discrepancy reduced to the 2 σ level or less → $\chi^{(r)}_{\text{KTeV}}(M_\rho) = 4.5 \pm 1.0$
- LMD model (*Knecht et al.*, PRL (1999) 83)
  $\chi^{(r)}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$

NLO radiative corrections in the QED sector did not solve the discrepancy

→ back to LO, but use different model

THS model for PVV correlator

1) Ansatz for Pseudoscalar-Vector-Vector (PVV) correlator

- Two-Hadron-Saturation (THS) - 2 meson multiplets per channel

$$\Pi^{\text{THS}}(r^2; p^2, q^2) \sim \frac{1}{r^2(r^2 - M_P^2)} \frac{P(r^2; p^2, q^2)}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)(q^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)}$$

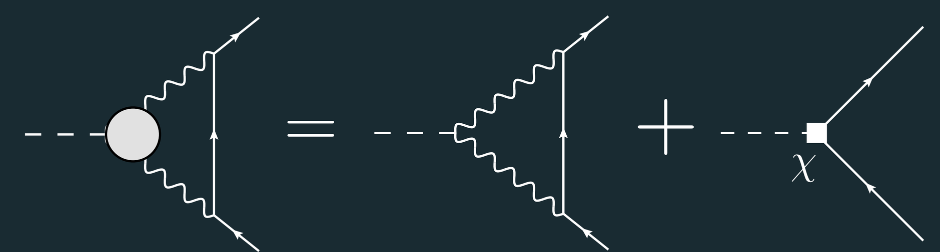
- in numerator stands general polynomial symmetrical in p^2 and q^2
  → correlator must drop at large momenta
  → 22 free parameters

$$P(r^2; p^2, q^2) = c_0 p^2 q^2 + c_1[(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \dots$$

2) Use high- and low-energy limits to constrain the parameters

- Operator product expansion (OPE)
- Brodsky–Lepage (BL) quark counting rules
- chiral anomaly

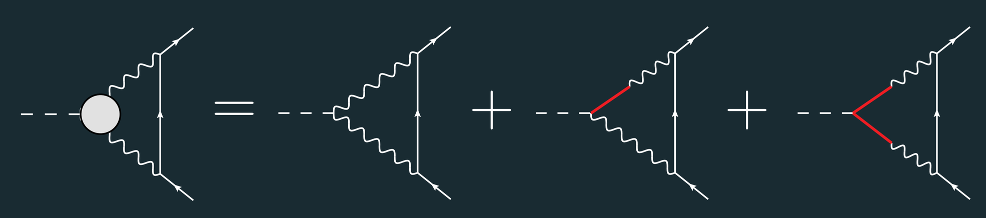
Resonances



Chiral Perturbation Theory (χPT)

⇕

Resonance Chiral Theory (RχT)



VMD and LMD models

Examples of other approaches

- Vector-Meson Dominance (VMD)

$$\mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2, q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4}{(p^2 - M_{V_1}^2)(q^2 - M_{V_1}^2)}\right]$$

→ violates OPE: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2, q^2) \not\sim \frac{1}{q^2}$, $q^2 \to -\infty$

- Lowest-Meson Dominance (LMD)

$$\mathcal{F}^{\text{LMD}}_{\pi^0\gamma^*\gamma^*}(p^2, q^2) = \mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2, q^2)\left\{1 - \frac{4\pi^2 F^2 (p^2 + q^2)}{N_c M_{V_1}^4}\right\}$$

→ violates BL: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2) \not\sim \frac{1}{q^2}$, $q^2 \to -\infty$

- none of the models used two meson multiplets in both channels
  → vector and pseudoscalar

THS and $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ form factor

Form factor is in general related to PVV correlator as

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2, q^2) \sim \lim_{r^2 \to 0} r^2 \Pi(r^2; p^2, q^2)$$

→ in our case complicated, but with only one free parameter

$$\mathcal{F}^{\text{THS}}_{\pi^0\gamma^*\gamma^*}(p^2, q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)(q^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)}\right] \times \left\{1 + \frac{\kappa}{2N_c} \frac{p^2 q^2}{(4\pi F)^4} - \frac{4\pi^2 F^2 (p^2 + q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6 + \frac{p^2 q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

κ determined from fit to ω-π transition form factor measurements

$$\kappa = 21 \pm 3$$

$M_{V_1} \sim \rho, \omega$ vector-meson mass
$M_{V_2} \sim$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740]\,\text{MeV}$$

Results

Theoretical prediction within THS model

$$B^{\text{THS}}(\pi^0 \to e^- e^+(\gamma),\, x_D > 0.95) = (5.8 \pm 0.2) \times 10^{-8}$$

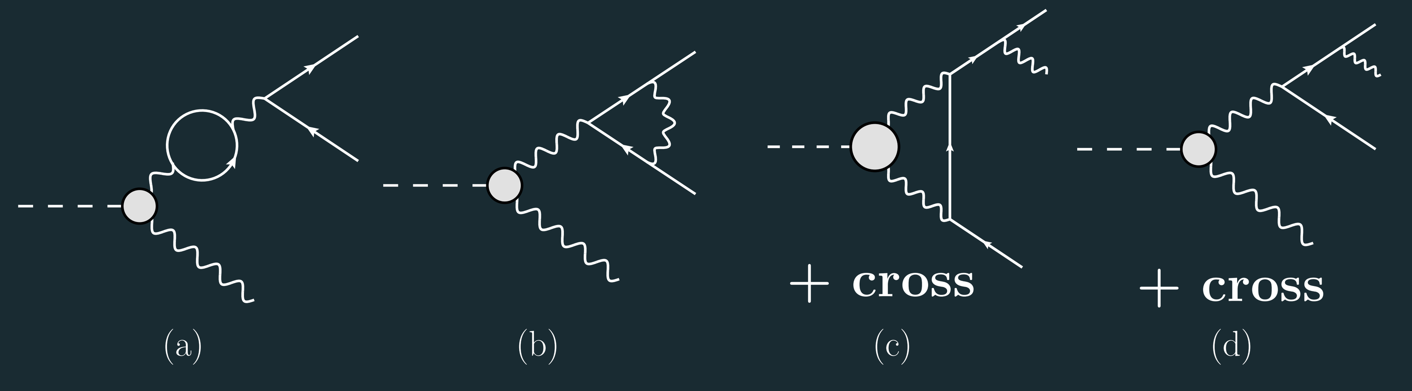
- recall experimental value: $B^{\text{KTeV}} = (6.44 \pm 0.33) \times 10^{-8}$
  → disagreement at the level of only 1.8 σ
- matching on LO χPT gives $\chi^{(r)}_{\text{THS}}(M_\rho) = 2.2 \pm 0.7$
- if KTeV result confirmed → two scenarios are conceivable:
  a) some aspects of the THS approach not well-suited for $\pi^0 \to e^+ e^-$
  b) beyond-Standard Model physics influences this decay significantly
- under present circumstances the current discrepancy is inconclusive

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0 \to e^+ e^-(\gamma),\, x > 0.95)}{\Gamma(\pi^0 \to e^+ e^- \gamma(\gamma),\, x > 0.2319)}\right|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

→ Dalitz decay comes into play

Dalitz decay radiative corrections



- corrections to the Dalitz plot in the form of a table of values (*Mikaelian and Smith*, PRD (1972) 5)
- new calculations motivated by needs of NA48/NA62 experiments
  → measure the slope a of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$
- unlike before no approximation was used
  → can be used also for related decays $\eta \to \ell^+ \ell^- \gamma$ etc.
- C++ code returns the correction for any given x and y
  → propagated into simulation software of NA62 experiment

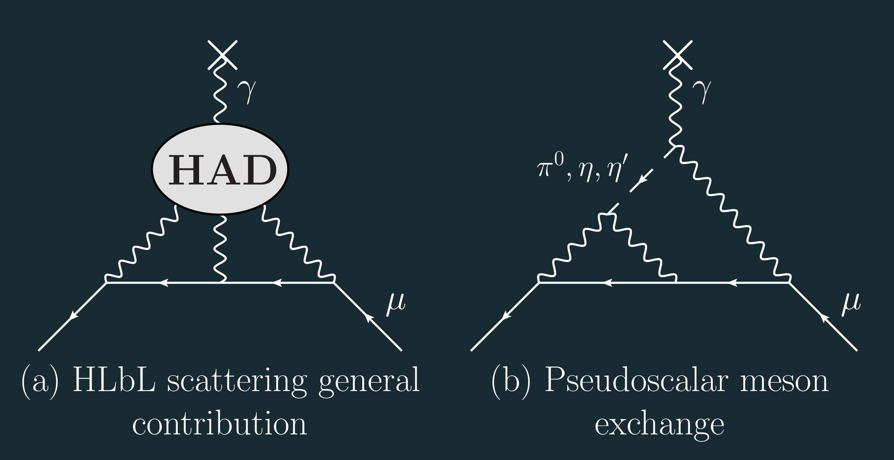
Outlook

Pseudoscalar decays

- $\chi^{(r)}$ universal for $P \to l^+ l^-$ processes up to corrections $\mathcal{O}(m_l^2/\Lambda^2_{\chi\text{PT}})$

Muon $g-2$

- hadronic light-by-light scattering
  → pseudoscalar meson exchange contribution requires hadron-physics input



(a) HLbL scattering general contribution — (b) Pseudoscalar meson exchange

Summary

All NLO QED radiative corrections for discussed processes are now available
→ can be taken into account in future experimental analyses

- $\pi^0 \to e^+ e^-$
  Vaško and Novotný, JHEP (2011) 1110
  TH, Kampf and Novotný, EPJC (2014) 74
- $\pi^0 \to e^+ e^- \gamma$
  TH, Kampf and Novotný, PRD (2015) 92

THS model for $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2, q^2)$

- phenomenologically successful
- satisfies all main theoretical constraints
- *TH and S. Leupold*, EPJC (2015) 75

Altogether, we get reasonable SM prediction
→ differs from KTeV by 1.8 σ

Supported by the grant GAUK 700214