Six-meson amplitude in QCD-like theories

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Introduction

In low-energy region, we cannot study perturbatively the interactions of hadrons directly from QCD
↪ alternative approaches → Chiral perturbation theory (ChPT)
Weinberg, Phys.A 96, (1979), *Gasser and Leutwyler*, Ann.Ph.158 (1984)

Many observables are known in ChPT to a high loop order
↪ only recently it has become of interest to calculate the six-pion amplitude at low energies after it has been estimated using lattice QCD
Blanton et al., PRL 124 (2020), JHEP 10 (2021), *Fischer et al.*, EPJC 81 (2021), *Hansen et al.*, PRL 126 (2021), *Brett et al.*, PRD 104 (2021)

The six-pion amplitude at tree level was first done using current algebra methods
e.g. *Osborn*, Lett.N.Cim.2 (1969)
It has been redone with Lagrangian methods many times, not known to one-loop order
e.g. *Low et al.*, JHEP 11 (2019), *Bijnens et al.*, JHEP 11 (2019)

We have therefore calculated at NLO the six-pion (and most recently the six-meson) amplitude (as well as the four-meson amplitude, meson mass and decay constant)
↪ within QCD-like theories with global symmetry and breaking patterns
$\mathrm{SU}(n) \times \mathrm{SU}(n)/\,\mathrm{SU}(n)$ (extending the QCD case), $\mathrm{SU}(2n)/\,\mathrm{SO}(2n)$, and $\mathrm{SU}(2n)/\,\mathrm{Sp}(2n)$

The relation to the measurement of the lattice is nontrivial to implement given
↪ complexity of the three-body finite volume calculations
↪ subtraction of the two-body rescatterings involved
Hansen et al., PRD 90 (2014), PRD 92 (2015), *Hammer et al.*, JHEP 09 (2017), JHEP 10 (2017), *Mai et al.*, EPJA 53 (2017), PRL 122 (2019), *Romero-López et al.*, JHEP 02 (2021), *Blanton et al.*, PRD 102 (2020)

Relevant Lagrangian for meson–meson scattering at NLO, $\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)}$

$$
\begin{aligned}
\mathcal{L}^{(2)} &= \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \\
\mathcal{L}^{(4)} &= L_0\langle u_\mu u_\nu u^\mu u^\nu\rangle + L_1\langle u_\mu u^\mu\rangle\langle u_\nu u^\nu\rangle \\
&+ L_2\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle \\
&+ L_4\langle u_\mu u^\mu\rangle\langle \chi_+\rangle + L_5\langle u_\mu u^\mu \chi_+\rangle \\
&+ L_6\langle \chi_+\rangle^2 + L_7\langle \chi_-\rangle^2 + \tfrac{1}{2}L_8\langle \chi_+^2 + \chi_-^2\rangle
\end{aligned}
$$

$$
\begin{aligned}
u_\mu &\equiv i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger) \\
\chi_\pm &\equiv u^\dagger \chi u^\dagger \pm u\chi^\dagger u \qquad\qquad u = \exp\left(\frac{i}{\sqrt{2}F}\,\phi^a t^a\right)
\end{aligned}
$$

For our application and with all the mesons having the same mass M, we put $\chi = M^2\mathbb{1}$

F, M: bare meson decay constant and mass

UV-finite parts of the coefficients (low-energy constants) L_i are free parameters in the theory
$\hookrightarrow$ both UV divergent and finite parts

Chiral Perturbation Theory for QCD-like theories at NLO

$$\begin{aligned}\mathcal{L}^{(4)} &= L_0\langle u_\mu u_\nu u^\mu u^\nu\rangle + L_1\langle u_\mu u^\mu\rangle\langle u_\nu u^\nu\rangle \\ &+ L_2\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle \\ &+ L_4\langle u_\mu u^\mu\rangle\langle \chi_+\rangle + L_5\langle u_\mu u^\mu \chi_+\rangle \\ &+ L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + \tfrac{1}{2}L_8\langle\chi_+^2 + \chi_-^2\rangle\end{aligned}$$

to NLO, one writes

$$L_i = L_i^{\mathsf{r}} - \frac{1}{16\pi^2}\frac{\Gamma_i}{2}\left(\frac{2}{4-d} - \gamma_{\mathsf{E}} + \log 4\pi - \log\mu^2 + 1\right)$$

From studying the meson mass, decay constant and the four-meson amplitude (Γ_7 from six-meson)

$$\Gamma_0 = \tfrac{1}{48}(n+4\xi) \qquad \Gamma_5 = \tfrac{n}{8}$$

$$\Gamma_1 = \tfrac{1}{16\zeta} \qquad \Gamma_6 = \tfrac{1}{16\zeta} + \tfrac{1}{8\zeta^2 n^2}$$

$$\Gamma_2 = \Gamma_4 = \tfrac{1}{8\zeta} \qquad \Gamma_7 = 0$$

$$\Gamma_3 = \tfrac{1}{24}(n-2\xi) \qquad \Gamma_8 = \tfrac{1}{16}(n+\xi-\tfrac{4}{\zeta n})$$

$$\xi \equiv \begin{cases} 0 & [\mathrm{SU}] \\ \pm 1 & [\mathrm{S}^{\mathrm{O}}_{\mathrm{p}}]\end{cases}$$

$$\zeta = 1+\xi^2 = \begin{cases} 1 & [\mathrm{SU}] \\ 2 & [\mathrm{S}^{\mathrm{O}}_{\mathrm{p}}]\end{cases}$$

Each meson ϕ^a carries flavor index a
$\hookrightarrow$ in the amplitude carried by G/H generator residing in flavor-space trace

Pair of fields is Wick-contracted $\longrightarrow$ corresponding flavor indices summed over
$\hookrightarrow$ resulting expressions are evaluated using the Fierz identities

$\mathrm{SU}(n)$

$$\langle t^a A\rangle\langle t^a B\rangle = \langle AB\rangle - \tfrac{1}{n}\langle A\rangle\langle B\rangle$$
$$\langle t^a A t^a B\rangle = \langle A\rangle\langle B\rangle - \tfrac{1}{n}\langle AB\rangle$$

$$\xi \equiv \begin{cases} 0 & [\,\mathrm{SU}\,] \\ \pm 1 & [\,\mathrm{S}^{\mathrm{O}}_{\mathrm{P}}\,] \end{cases}$$

$\mathrm{S}^{\mathrm{O}}_{\mathrm{P}}(2n)$

$$\zeta = 1 + \xi^2 = \begin{cases} 1 & [\,\mathrm{SU}\,] \\ 2 & [\,\mathrm{S}^{\mathrm{O}}_{\mathrm{P}}\,] \end{cases}$$

$$\langle t^a A\rangle\langle t^a B\rangle = \tfrac{1}{2}\big[\langle AB\rangle + \langle AB^\dagger\rangle\big] - \tfrac{1}{2n}\langle A\rangle\langle B\rangle$$
$$\langle t^a A t^a B\rangle = \tfrac{1}{2}\big[\langle A\rangle\langle B\rangle \pm \langle AB^\dagger\rangle\big] - \tfrac{1}{2n}\langle AB\rangle$$

$\hookrightarrow$ the only source of formal dissimilarity between the amplitudes for the different cases (Lagrangians formally identical)

Four-meson amplitude

On-shell amplitude in general

p_i, $i = 1, \dots, 4$ meson incoming four-momenta, $\sum p_i = 0$

b_i flavours

Mandelstam variables

$s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$, $u = (p_2 + p_3)^2$, $s + t + u = 4M^2$

The amplitude is conventionally decomposed as

$$\begin{aligned} A_{4\pi}(s,t,u) = {} & \big(\langle t^{b_1} t^{b_2} t^{b_3} t^{b_4}\rangle + \langle t^{b_4} t^{b_3} t^{b_2} t^{b_1}\rangle\big) B(s,t,u) \\ & + \big(\langle t^{b_1} t^{b_3} t^{b_4} t^{b_2}\rangle + \langle t^{b_2} t^{b_4} t^{b_3} t^{b_1}\rangle\big) B(t,u,s) \\ & + \big(\langle t^{b_1} t^{b_4} t^{b_2} t^{b_3}\rangle + \langle t^{b_3} t^{b_2} t^{b_4} t^{b_1}\rangle\big) B(u,s,t) \\ & + \delta^{b_1 b_2} \delta^{b_3 b_4} C(s,t,u) + \delta^{b_1 b_3} \delta^{b_2 b_4} C(t,u,s) \\ & + \delta^{b_1 b_4} \delta^{b_2 b_3} C(u,s,t) \end{aligned}$$

↪ structure follows from requiring invariance under the unbroken group, Bose symmetry and charge conjugation for SU

↪ under $\mathrm{S}_{\mathrm{p}}^{\mathrm{O}}$, $\langle t^a t^b t^c t^d\rangle = \langle t^d t^c t^b t^a\rangle$ without relying on charge conjugation

↪ functions satisfy $B(s,t,u) = B(u,t,s)$ and $C(s,t,u) = C(s,u,t)$

Amplitude decomposition

Flavor stripping

$$A_{k\pi}(p_1, b_1; p_2, b_2; \ldots; p_k, b_k) = \sum_R \sum_\sigma A_R^\sigma(p_1, \ldots, p_k)\mathcal{F}_R^\sigma(b_1, \ldots, b_k)$$

Deorbiting

$$A_R(p_1, \ldots, p_k) = \sum_{\sigma \in \mathbb{Z}_R^{+\mathrm{TR}}} \tilde{A}_R(p_{\sigma_1}, \ldots, p_{\sigma_k})$$

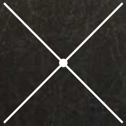
Group-universal form

$$A = \left\{ \mathcal{A}^{(1)} + \xi \mathcal{A}^{(\xi)} + \xi^2 \mathcal{A}^{(\xi^2)} + \frac{\mathcal{A}^{(\zeta)}}{\zeta} \right\}_{n \to \zeta n}$$

$\hookrightarrow$ clearly redundant $\to$ 3 amplitudes expressed as a combination of 4 subamplitudes

Combined together $\to$ amplitude formulated using very concise subamplitudes $\tilde{\mathcal{A}}_R^{(i)}$
$\hookrightarrow$ many of these are actually zero

The leading-order $\mathcal{O}(p^2)$ amplitude stems from a single diagram

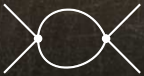


$\hookrightarrow$ schematically $A_{4\pi}^{(\text{LO})} = \mathcal{M}_{\text{LO}}^{(2)}|_{\text{on-shell}}$

Related LO subamplitude (with LO relations $M \to M_\pi$ and $F \to F_\pi$)

$$\mathcal{A}_{\{4\}}^{(\text{LO},1)} = 8\tilde{\mathcal{A}}_{\{4\}}^{(\text{LO},1)} = -\frac{t - 2M_\pi^2}{2F_\pi^2}$$

At NLO, one-loop diagrams (two topologies of 4 one-loop diagrams in total) and a counterterm



(a) 3×

(b) 1×

(c) 1×

+ NLO field renormalization, and mass and decay-constant redefinitions applied to the LO graph
↪ schematically $A_{4\pi}^{(\mathrm{NLO})} = \mathcal{M}_{\text{1-loop}} + \mathcal{M}_{\mathrm{CT}} + 4(Z^{1/2}-1)\mathcal{M}_{\mathrm{LO}}^{(2)} + \mathcal{M}_{\mathrm{LO}}^{(4)}$
The Z factor is related to the meson self-energy Σ

$$\frac{1}{Z} = 1 - \frac{\partial \Sigma(p^2)}{\partial p^2}\bigg|_{p^2=M_\pi^2}$$

Standard relations $M_\pi^2 = M^2 + \overline{\Sigma}$, $F_\pi = F(1+\delta F)$ give the substitutions at the given order

$$M^k \to M_\pi^k\left(1 - \frac{k}{2}\frac{\overline{\Sigma}}{M_\pi^2}\right), \qquad \overline{\Sigma} = \frac{M_\pi^4}{F_\pi^2}\left\{-8\left[L_5^{\mathrm{r}} - 2L_8^{\mathrm{r}} + n\zeta(L_4^{\mathrm{r}} - 2L_6^{\mathrm{r}})\right] + \left(\frac{1}{\zeta n} - \frac{\xi}{2}\right)L\right\} + \mathcal{O}\left(\frac{1}{F_\pi^4}\right)$$

$$\frac{1}{F^k} \to \frac{1}{F_\pi^k}(1 + k\delta F)\,, \qquad \delta F = \frac{M_\pi^2}{F_\pi^2}\left[4(L_5^{\mathrm{r}} + n\zeta L_4^{\mathrm{r}}) - \frac{n}{2}L\right] + \mathcal{O}\left(\frac{1}{F_\pi^4}\right)$$

Parametrization-independent and UV-finite result

$$F_\pi^4 \tilde{\mathcal{A}}^{(\mathsf{NLO},1)}_{\{2,2\}} = \frac{M_\pi^4}{4n^2}\Big\{\bar{J}(s) - (L+\kappa)\Big\} + \frac{u(u-t)}{2}L_2^{\mathrm{r}} + 4M_\pi^4(L_1^{\mathrm{r}} - L_4^{\mathrm{r}} + L_6^{\mathrm{r}}) + \frac{s^2}{4}(4L_1^{\mathrm{r}} + L_2^{\mathrm{r}}) - 2M_\pi^2 s(2L_1^{\mathrm{r}} - L_4^{\mathrm{r}})$$

$$F_\pi^4 \tilde{\mathcal{A}}^{(\mathsf{NLO},\zeta)}_{\{2,2\}} = \frac{s^2\bar{J}(s)}{32} + \frac{\bar{J}(u)}{16}\Big\{(u-2M_\pi^2)^2\Big\} - \frac{L+\kappa}{64}\Big\{3s^2 - 2u(t-u)\Big\}$$

and similarly for $\tilde{\mathcal{A}}^{(\mathsf{NLO},1)}_{\{4\}}$, $\tilde{\mathcal{A}}^{(\mathsf{NLO},\xi)}_{\{4\}}$, $\tilde{\mathcal{A}}^{(\mathsf{NLO},\zeta)}_{\{4\}}$

$\hookrightarrow$ different form but identical to *Bijnens and Lu*, JHEP 03 (2011)

Above we used

$$\kappa = \frac{1}{16\pi^2}\,, \quad L \equiv \kappa \log\frac{M_\pi^2}{\mu^2}\,, \quad \bar{J}(q^2) \equiv \kappa\left(2 + \beta\log\frac{\beta-1}{\beta+1}\right), \quad \beta = \sqrt{1 - \frac{4M_\pi^2}{q^2}}$$

Six-meson amplitude

Decomposition in terms of six flavor labels and momenta

$$
\begin{aligned}
&A_{6\pi}(p_1,\dots,p_6)\\
&\quad = \sum_{\mathcal{S}_6}\Big\{\tfrac{1}{12}\Big[\langle t^{b_1}\cdots t^{b_6}\rangle + \langle t^{b_6}\cdots t^{b_1}\rangle\Big]D(p_1,\dots,p_6)\\
&\qquad\qquad + \tfrac{1}{16}\delta^{b_1 b_2}\Big[\langle t^{b_3}\cdots t^{b_6}\rangle + \langle t^{b_6}\cdots t^{b_3}\rangle\Big]E(p_1,\dots,p_6)\\
&\qquad\qquad + \tfrac{1}{36}\Big[\langle t^{b_1}t^{b_2}t^{b_3}\rangle\langle t^{b_4}t^{b_5}t^{b_6}\rangle + \langle t^{b_3}t^{b_2}t^{b_1}\rangle\langle t^{b_6}t^{b_5}t^{b_4}\rangle\Big]F(p_1,\dots,p_6)\\
&\qquad\qquad + \tfrac{1}{48}\delta^{b_1 b_2}\delta^{b_3 b_4}\delta^{b_5 b_6}G(p_1,\dots,p_6)\Big\}
\end{aligned}
$$

$\hookrightarrow$ $\mathcal{S}_6$ represents the $6! = 720$ permutations of $\{1,\dots,6\}$

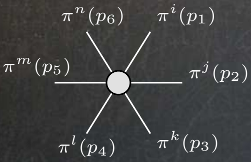
$\hookrightarrow$ symmetry factors $\leftrightarrow$ how many permutations leave the flavor structure unchanged

$\hookrightarrow$ D, E, F and G summed over 60, 45, 20 and 15 distinct permutations (B and C summed over 3)

Six-meson amplitude

4 mesons $\to$ 3 channels/permutations/ways to distribute 4 mesons in 2 pairs

6 mesons $\to$ 10 ways in 2 groups of three (P_{10})
$\hookrightarrow$ 15 ways in 3 pairs (P_{15})



The full six-meson amplitude at $\mathcal{O}(p^4)$

$$A_{6\pi} = A_{6\pi}^{(\text{pole})} + A_{6\pi}^{(\text{non-pole})}$$

$\hookrightarrow$ (a) only contributes to $A_{6\pi}^{(\text{non-pole})}$

$\hookrightarrow$ (b) contributes to both the pole and non-pole parts

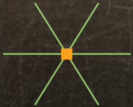
$A_{6\pi}^{(\text{pole})}$ can be written in terms of the four-meson amplitude and $A_{6\pi}^{(\text{non-pole})}$ is the remainder

(a) 1×

(b) 10×

Six-meson amplitude

Feynman diagrams of relevant topologies



(a) 1×

(b) 1×

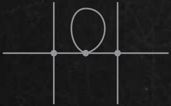
(c) 15×

(d) 20×

(e) 20×

(f) 60×

(g) 10×



(h) 10×

(i) 15×

$$A_{6\pi}^{\text{(pole)}} = \sum_{P_{10},b_\circ} A_{4\pi}(p_i, b_i; p_j, b_j; p_k, b_k; -p_{ijk}, b_\circ) \frac{-1}{p_{ijk}^2 - M_\pi^2} A_{4\pi}(p_\ell, b_\ell; p_m, b_m; p_n, b_n; p_{ijk}, b_\circ)$$

↪ residue at the pole unique, off-shell extrapolation away from $p_{ijk}^2 \equiv (p_i + p_j + p_k)^2 = M_\pi^2$ not
$A_{4\pi}(p_i, b_i; p_j, b_j; p_k, b_k; -p_{ijk}, b_\circ)$ is the four-meson amplitude with one leg off-shell

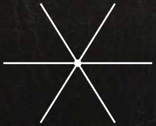
↪ $s = (p_i + p_j)^2$, $t = (p_i + p_k)^2$ and $u = (p_j + p_k)^2$, although now $s + t + u = 3M_\pi^2 + p_{ijk}^2$

We have chosen a particular form for the off-shell four-meson subamplitude
↪ other off-shell extrapolations are possible and will lead to a different $A_{6\pi}^{\text{(non-pole)}}$
↪ independent of the parametrization used
  ↪ also $A_{4\pi}$ and, consequently, the respective parts $A_{6\pi}^{\text{(pole)}}$ and $A_{6\pi}^{\text{(non-pole)}}$ by definition
↪ the way how the contributions from the one-particle irreducible and reducible diagrams are distributed within the final result is parametrization dependent

At LO simple expressions

$$\mathcal{A}_{\{6\}}^{(\text{LO, non-pole, }1)} = \frac{p_1 \cdot p_3 + p_3 \cdot p_5 + p_5 \cdot p_1}{2F_\pi^4}$$

$$\mathcal{A}_{\{3,3\}}^{(\text{LO, non-pole, }1)} = \frac{M_\pi^2 - p_1 \cdot p_2 - p_1 \cdot p_3 - p_2 \cdot p_3}{2nF_\pi^4}$$



(a) 1×

(b) 10×

The main new result is the next-order six-meson non-pole amplitude
↪ decomposed in 3 mutually orthogonal directions

$$
\begin{aligned}
F_\pi^6 \tilde{A}^{(\mathsf{NLO},1)}_{\{6\}} &= \tfrac{M_\pi^4}{4n}\Big\{\bar{J}(p_1,p_2) - L - \kappa\Big\} \\
&+ \tfrac{M_\pi^4}{8n}\Big\{2C_{11}(p_1,\ldots,p_6) + 2C(p_1,\ldots,p_6)\big[s_7 - M_\pi^2\big] + C(p_1,p_6,p_2,p_5,p_3,p_4)\big[s_8 - 2s_6 + s_9\big]\Big\} \\
&- L_0^\mathrm{r}\Big\{2M_\pi^4 + 4M_\pi^2(s_7 - 2s_1) + s_1(s_1 + 2s_4 + 3s_5 + 2s_6 - 3s_7) - s_7(3s_2 + 2s_3 - s_7 - s_9)\Big\} \\
&- \tfrac{1}{4}L_3^\mathrm{r}\Big\{7M_\pi^4 - 2M_\pi^2(7s_1 - 4s_7) + s_1(2s_1 + 2s_4 + 2s_5 - 3s_7 - 3s_9) + s_7^2\Big\} \\
&+ M_\pi^2 L_5^\mathrm{r}\Big\{2M_\pi^2 - 2s_1 + s_7\Big\} - 2M_\pi^4 L_8^\mathrm{r}
\end{aligned}
$$

and other nonzero

$$\tilde{A}^{(\mathsf{NLO},\xi)}_{\{6\}},\ \tilde{A}^{(\mathsf{NLO},\zeta)}_{\{6\}},\ \tilde{A}^{(\mathsf{NLO},1)}_{\{2,4\}},\ \tilde{A}^{(\mathsf{NLO},\zeta)}_{\{2,4\}},\ \tilde{A}^{(\mathsf{NLO},1)}_{\{3,3\}},\ \tilde{A}^{(\mathsf{NLO},\xi)}_{\{3,3\}},\ \tilde{A}^{(\mathsf{NLO},\xi^2)}_{\{3,3\}},\ \tilde{A}^{(\mathsf{NLO},\zeta)}_{\{3,3\}},\ \tilde{A}^{(\mathsf{NLO},1)}_{\{2,2,2\}}$$

Large number of kinematic invariants → reduction to master integrals (scalar triangle integrals) leads to an enormous expression
↪ we have chosen a redundant basis of integrals that have good symmetry properties

Results are rather lengthy, but can be written in a relatively compact way
↪ see paper arXiv:2207.02234

We choose a symmetric $3 \to 3$ scattering configuration given by

$$p_1 = \Big(E_p, p, 0, 0\Big) \qquad p_4 = \Big(-E_p, 0, 0, p\Big)$$

$$p_{2,3} = \left(E_p, -\frac{1}{2}p, \pm\frac{\sqrt{3}}{2}p, 0\right) \qquad p_{5,6} = \left(-E_p, \pm\frac{\sqrt{3}}{2}p, 0, -\frac{1}{2}p\right)$$

We use following numerical inputs:

$$M_\pi = 0.139570\,\text{GeV} \qquad L_1^{\mathrm{r}} = 1.0\times 10^{-3} \qquad L_5^{\mathrm{r}} = 1.2\times 10^{-3}$$

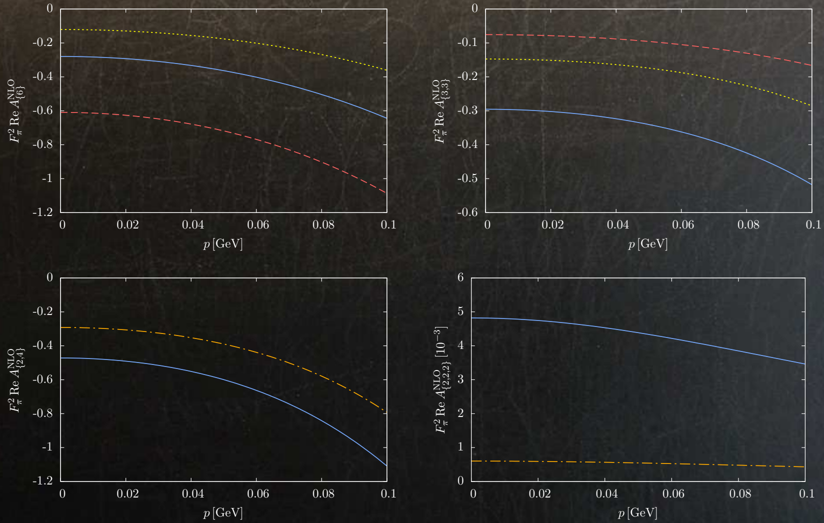
$$F_\pi = 0.0927\,\text{GeV} \qquad L_2^{\mathrm{r}} = 1.6\times 10^{-3} \qquad L_6^{\mathrm{r}} = 0$$

$$\mu = 0.77\,\text{GeV} \qquad L_3^{\mathrm{r}} = -3.8\times 10^{-3} \qquad L_7^{\mathrm{r}} = -0.3\times 10^{-3}$$

$$n = 3 \qquad \bar{L}_4^{\mathrm{r}} = 0 \qquad L_8^{\mathrm{r}} = 0.5\times 10^{-3}$$

For $n = 3$, we use $L_0^{\mathrm{r}} = 0$

Bijnens, Ecker, ARNPS 64 (2014)


0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
$F_\pi^2\,\mathrm{Re}\,A^{\mathrm{NLO}}_{\{6\}}$
0
0.02
0.04
0.06
0.08
0.1
$p\,[\mathrm{GeV}]$
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
$F_\pi^2\,\mathrm{Re}\,A^{\mathrm{NLO}}_{\{3,3\}}$
0
0.02
0.04
0.06
0.08
0.1
$p\,[\mathrm{GeV}]$
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
$F_\pi^2\,\mathrm{Re}\,A^{\mathrm{NLO}}_{\{2,4\}}$
0
0.02
0.04
0.06
0.08
0.1
$p\,[\mathrm{GeV}]$
6
5
4
3
2
1
0
$F_\pi^2\,\mathrm{Re}\,A^{\mathrm{NLO}}_{\{2,2,2\}}\,[10^{-3}]$
0
0.02
0.04
0.06
0.08
0.1
$p\,[\mathrm{GeV}]$

Summary

We calculated the meson mass, decay constant, the four-meson and six-meson amplitudes to NLO in the QCD-like theories

$\hookrightarrow$ single NLO Lagrangian consistent with three(-quark)-flavour ChPT

Our main result is the six-meson amplitude
$\hookrightarrow$ split in pole and non-pole parts

The pole part employs the off-shell four-meson amplitude
$\hookrightarrow$ consistent with *Bijnens and Lu*, JHEP 03 (2011)

The non-pole part can be written in terms of 10 subamplitudes
$\hookrightarrow$ fairly compact expressions for deorbited stripped group-universal subamplitudes $\tilde{\mathcal{A}}_R^{(i)}$
 $\hookrightarrow$ allowed also by choosing the basis of triangle loop integrals with high symmetry
$\hookrightarrow$ NLO corrections sizable

Outlook

Work in progress
$\hookrightarrow$ combine our results with the methods for extracting three-body scattering from finite volume in lattice QCD

Might be of interest for the amplitude community

More details in arXiv:2206.14212

$\hookrightarrow$ preceding work: PRD 104 (2021) 054046, arXiv:2107.06291