



# Radiative corrections for the decay $\Sigma^0 \rightarrow \Lambda e^+ e^-$

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How are the building blocks **distributed** inside of the composite objects (nucleons)?

- electron–nucleon scattering
  - electromagnetic form factors
    - ↪ used to explore intrinsic structure of nucleons
  - we deduce low-energy quantities
    - ↪ electric charge, magnetic moment, electric and magnetic **radii**

**Replacement** of down quarks (of a nucleon/ $\Delta$ ) by **strange** quark(s)?

- played role in revealing quarks as building blocks of nucleons and hadrons in general
- close relation among the intrinsic structures of hyperons and nucleons
  - hyperon electromagnetic and transition form factors contain **complementary** information to the nucleon (and  $\Delta$ ) ones





Experimental knowledge of hyperons rather **limited**

- electric charges and magnetic moments known
- **unstable**  $\rightarrow$  electron scattering rather difficult

Form factors

- **high** energies
  - $e^+e^-$  scattering to hyperon + antihyperon  
 $\leftrightarrow$  both direct and transition form factors accessible
- **low** energies
  - **Dalitz decay**  $Y \rightarrow Y'e^+e^-$
  - possibly high statistics in **future** at FAIR (PANDA:  $p\bar{p}$ , HADES:  $pp$ )

Dalitz decays in baryon-octet sector?

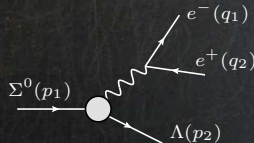
# Dalitz decay of $\Sigma^0$

## Introduction



Dalitz decay  $\Sigma^0 \rightarrow \Lambda e^+ e^-$

- $e^+e^-$  invariant mass only up to  $M_\Sigma - M_\Lambda \simeq 77 \text{ MeV}$
- provides electric and magnetic transition form factors of  $\Sigma^0 \rightarrow \Lambda$  transition
- extracting transition radii challenging
  - high-precision measurement required
  - competing with QED radiative corrections



Predictions of electric and magnetic radii

- *Kubis and Meissner, EPJC 18 (2001)*
- *Granados, Leupold and Perotti, EPJA 53 (2017)*



$\Sigma^0 \Lambda \gamma$  vertex:  $\langle 0 | j^\mu | \Sigma^0 \bar{\Lambda} \rangle = e \bar{v}_\Lambda(\vec{p}_2) G^\mu(p_1 + p_2) u_\Sigma(\vec{p}_1)$ , with

$$G^\mu(q) \equiv \left[ \gamma^\mu - (M_\Sigma - M_\Lambda) \frac{q^\mu}{q^2} \right] G_1(q^2) - \frac{i \sigma^{\mu\nu} q_\nu}{M_\Sigma + M_\Lambda} G_2(q^2)$$

Define magnetic and electric form factors

$$G_M(q^2) \equiv G_1(q^2) + G_2(q^2) = \kappa \left( 1 + \frac{1}{6} \langle r_M^2 \rangle q^2 + \mathcal{O}(q^4) \right)$$

$$G_E(q^2) \equiv G_1(q^2) + \frac{q^2}{(M_\Sigma + M_\Lambda)^2} G_2(q^2) = \frac{1}{6} \langle r_E^2 \rangle q^2 + \mathcal{O}(q^4)$$

Matrix element squared dominated by the magnetic part

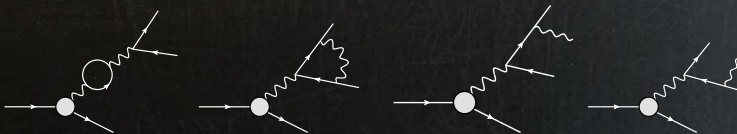
$$|\overline{\mathcal{M}^{\text{LO}}(x, y)}|^2 \simeq 2e^4 |G_M(\Delta_M^2 x)|^2 \frac{(1-x)}{x} \left( 1 + y^2 + \frac{\nu^2}{x} \right)$$

$$x \equiv \frac{(p_{e^+} + p_{e^-})^2}{(M_{\Sigma^0} - M_\Lambda)^2}, \quad y \equiv \frac{2 p_{\Sigma^0} \cdot (p_{e^+} - p_{e^-})}{\lambda^{\frac{1}{2}}(p_{\Sigma^0}^2, p_\Lambda^2, (p_{e^+} + p_{e^-})^2)}$$



Radiative corrections to the differential decay width in **soft-photon** approximation

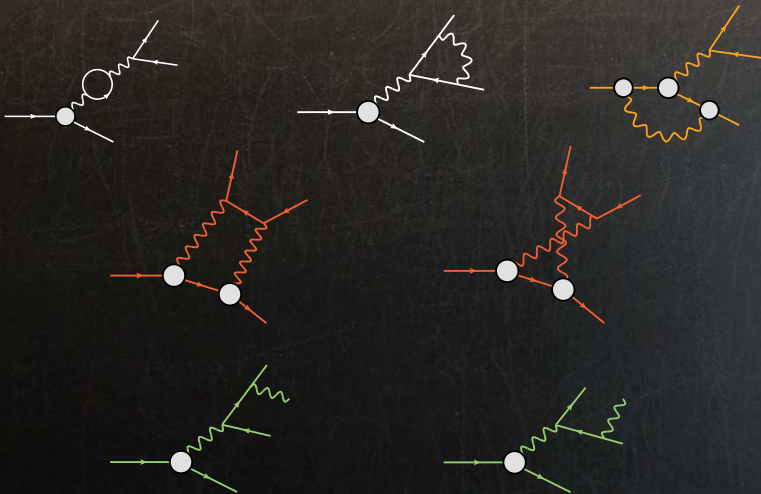
- *Sidhu and Smith, PRD 4 3344 (1971)*
  - no hard-photon corrections, low- $x$  region not covered
  - resulting corrections **negative** all over the Dalitz plot  
 $\hookrightarrow$  **total** correction known to be positive





*TH and Leupold, EPJC 80 (2020), arXiv:1911.02571*

- inclusive radiative corrections **beyond** the soft-photon approximation





Low-energy expansion of the form factors:

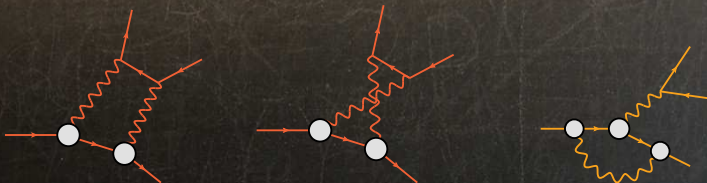
$$G_M((k + q_1 + q_2)^2) \simeq G_M((q_1 + q_2)^2) \left\{ 1 + \frac{1}{6} (r_M^2) [2k \cdot (q_1 + q_2)] \right\}$$

$$G_E((k + q_1 + q_2)^2) \simeq G_E((q_1 + q_2)^2) \left\{ 1 + \frac{2k \cdot (q_1 + q_2)}{(q_1 + q_2)^2} \right\}$$

Subsequently, integrate over the energy and emission angle of the bremsstrahlung photon

- radiative corrections for **inclusive** process





By loop-momenta-power counting, FFs required to regulate the UV region

1 $\gamma$ IR: UV convergence already achieved in the simplest case with **constant** FFs

- $G_E(q^2) = G_E(0) = 0$  and  $G_M(q^2) = G_M(0) = \kappa$

$$G_1(q^2) = \kappa \frac{q^2}{q^2 - M_V^2}, \quad G_2(q^2) = -\kappa \frac{M_V^2}{q^2 - M_V^2}$$

Ansatz satisfying high-energy constraints

$$G_1(q^2) = \kappa \left( 3 - \frac{M_V^2 \langle r_M^2 \rangle}{6} \right) \frac{q^2 M_V^4}{(q^2 - M_V^2)^3}, \quad G_2(q^2) = -\kappa \frac{M_V^6}{(q^2 - M_V^2)^3}$$

These contributions to the NLO decay width are found to be **negligible**



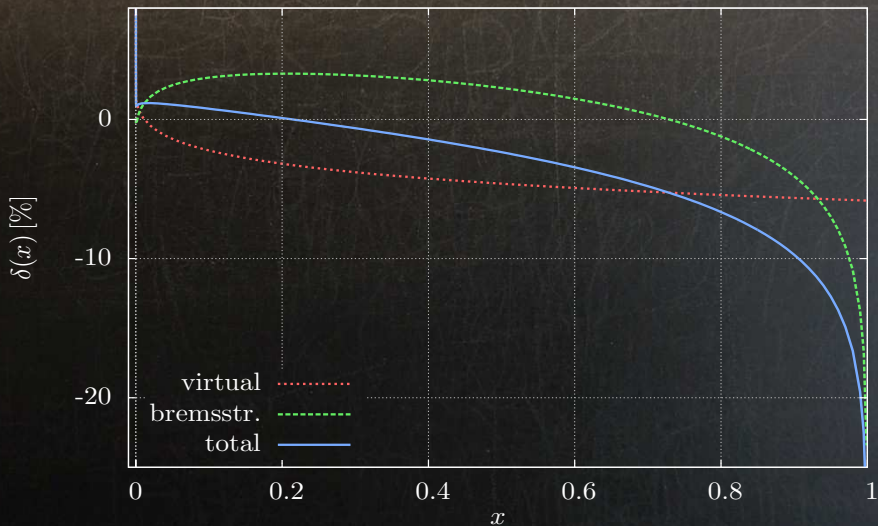
# Results

Corrections to two-fold differential decay width  $\delta(x, y)$  given in %

x \ y	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
0.01	2.49	2.43	2.30	2.16	2.02	1.85	1.63	1.32	0.80	-0.24	-8.33
0.02	2.65	2.59	2.47	2.32	2.16	1.97	1.72	1.37	0.80	-0.33	-6.00
0.03	2.69	2.64	2.52	2.38	2.21	2.01	1.75	1.37	0.77	-0.42	-5.96
0.04	2.68	2.64	2.53	2.39	2.22	2.01	1.74	1.35	0.73	-0.51	-6.08
0.05	2.65	2.61	2.51	2.37	2.21	2.00	1.72	1.32	0.68	-0.59	-6.24
0.06	2.61	2.57	2.48	2.35	2.19	1.98	1.70	1.29	0.63	-0.67	-6.41
0.07	2.56	2.53	2.44	2.31	2.15	1.95	1.66	1.25	0.58	-0.75	-6.58
0.08	2.50	2.47	2.39	2.27	2.12	1.91	1.63	1.21	0.53	-0.83	-6.75
0.09	2.44	2.42	2.34	2.22	2.07	1.87	1.59	1.16	0.47	-0.90	-6.91
0.10	2.38	2.35	2.28	2.17	2.03	1.83	1.54	1.12	0.42	-0.98	-7.07
0.15	2.04	2.02	1.97	1.89	1.76	1.57	1.30	0.86	0.14	-1.35	-7.81
0.20	1.67	1.66	1.63	1.56	1.45	1.29	1.02	0.58	-0.17	-1.71	-8.49
0.25	1.29	1.28	1.26	1.21	1.12	0.97	0.71	0.28	-0.48	-2.09	-9.13
0.30	0.89	0.89	0.88	0.84	0.77	0.64	0.39	-0.04	-0.82	-2.47	-9.76
0.35	0.47	0.47	0.47	0.45	0.40	0.28	0.04	-0.39	-1.17	-2.88	-10.4
0.40	0.03	0.03	0.04	0.04	0.00	-0.11	-0.34	-0.76	-1.56	-3.30	-11.0
0.45	-0.44	-0.43	-0.42	-0.41	-0.43	-0.53	-0.74	-1.16	-1.97	-3.75	-11.6
0.50	-0.94	-0.93	-0.90	-0.88	-0.89	-0.97	-1.18	-1.59	-2.41	-4.24	-12.3
0.55	-1.48	-1.47	-1.43	-1.40	-1.40	-1.47	-1.66	-2.07	-2.90	-4.77	-12.9
0.60	-2.07	-2.06	-2.02	-1.97	-1.95	-2.01	-2.19	-2.60	-3.44	-5.35	-13.6
0.65	-2.73	-2.71	-2.66	-2.61	-2.58	-2.62	-2.79	-3.20	-4.05	-6.00	-14.4
0.70	-3.48	-3.46	-3.40	-3.33	-3.29	-3.31	-3.48	-3.89	-4.75	-6.73	-15.3
0.75	-4.34	-4.32	-4.25	-4.17	-4.11	-4.13	-4.29	-4.69	-5.57	-7.59	-16.2
0.80	-5.38	-5.35	-5.28	-5.19	-5.11	-5.12	-5.27	-5.67	-6.56	-8.62	-17.4
0.85	-6.69	-6.66	-6.58	-6.47	-6.39	-6.38	-6.52	-6.93	-7.83	-9.92	-18.8
0.90	-8.50	-8.47	-8.38	-8.26	-8.16	-8.14	-8.27	-8.68	-9.60	-11.7	-20.7
0.95	-11.5	-11.5	-11.4	-11.3	-11.1	-11.1	-11.2	-11.7	-12.6	-14.7	-23.8
0.99	-18.4	-18.4	-18.2	-18.1	-18.0	-17.9	-18.1	-18.5	-19.4	-21.6	-30.7

# Results

Corrections to one-fold differential decay width





Integrate over the Dalitz plot (values from *Kubis and Meissner, EPJC 18 (2001)*)

$$R \equiv \frac{\Gamma(\Sigma^0 \rightarrow \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \rightarrow \Lambda \gamma)} = 5.541(2) \times 10^{-3}$$

↔ neglect **electric** form factor → **expansion** in **magnetic** form-factor slope  $a \equiv \frac{1}{6} \langle r_M^2 \rangle \Delta_M^2$

$$R = R_0 + aR_1 + \mathcal{O}(a^2)$$

+ higher-order corrections as additional uncertainty

$$R = [5.530(3) + 0.626(2)a] \times 10^{-3}$$

↔ consistent with the **NLO** result in *Sidhu and Smith, PRD 4 3344 (1971)*

$$R_{S\&S} = (5.532 + 0.627a) \times 10^{-3} [\approx 5.544 \times 10^{-3}]$$



Using present values for physical constants (using NLO expressions from *Sidhu and Smith (1971)*)

$$R_{S\&S}^{\text{new}} = (5.52975 + 0.62640a) \times 10^{-3}$$

Compares well with our result  $R = (5.52974 + 0.62640a) \times 10^{-3}$  (only electron loop in VP)



	LO	virt	BS  <sub>C</sub>	BS  <sub>D</sub>	total
$R_0 [10^{-3}]$	5.4838	-0.01667	-0.06443	0.12713	5.5298
$R_1 [10^{-3}]$	0.6189	-0.02006	0.00010	0.02747	0.6264
$\delta$ [%]	—	-0.310	-1.173	2.322	0.839

From the constraint  $\mathcal{B}(\Sigma^0 \rightarrow \Lambda\gamma) + \mathcal{B}(\Sigma^0 \rightarrow \Lambda e^+e^-) + \mathcal{B}(\Sigma^0 \rightarrow \Lambda\gamma\gamma) \simeq 1$  we find

$$\mathcal{B}(\Sigma^0 \rightarrow \Lambda\gamma) \simeq \frac{1}{1+R} = [99.4501(3) - 0.0619(2)a] \%$$

$$\mathcal{B}(\Sigma^0 \rightarrow \Lambda e^+e^-) \simeq \frac{R}{1+R} = [0.5499(3) + 0.0619(2)a] \%$$

Using conservative  $a = 0.02(2)$

$$\mathcal{B}(\Sigma^0 \rightarrow \Lambda\gamma) = 99.449(2) \%$$

$$\mathcal{B}(\Sigma^0 \rightarrow \Lambda e^+e^-) = 0.551(2) \%$$

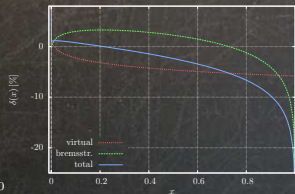
# Results

Correction to the form-factor slope

Estimate of size of correction to the magnetic form-factor slope  $a$

- take half of the slope of the curve in the **low- $x$**  region
- **farther** from the threshold ( $\nu^2 \ll x_0 \ll 1$ ):

$$\Delta a \equiv a_{(+\text{QED})} - a \simeq \frac{1}{2} \left. \frac{d\delta(x)}{dx} \right|_{x=x_0}$$



$a_{(+\text{QED})}$ : measured value implicitly containing the QED radiative correction

$$\frac{1}{2} \left. \frac{d\delta(x)}{dx} \right|_{x=x_0} \approx -3.5\%$$

- **bigger** than the estimate on the slope  $a \equiv \frac{1}{6} \langle r_M^2 \rangle \Delta_M^2$  itself ( $a \approx 1.8(3)\%$ )

Using **no** radiative corrections in the experimental analysis

- one expects “measured” radius  $\langle r_M^2 \rangle_{(+\text{QED})}$  to be **negative**

$$\langle r_M^2 \rangle_{(+\text{QED})} = \langle r_M^2 \rangle + \frac{6}{\Delta_M^2} \Delta a, \quad \text{with} \quad \frac{6}{\Delta_M^2} \Delta a \approx -35 \text{ GeV}^{-2}$$

( $\chi_{\text{PT}}$  :  $\langle r_M^2 \rangle = 18.5(2.6) \text{ GeV}^{-2}$ ); in general for hadronic radii:  $\langle r^2 \rangle \leq (1 \text{ fm})^2 \approx 25 \text{ GeV}^{-2}$



We calculated **complete inclusive** NLO QED radiative correction to the differential decay width  $\leftrightarrow \Sigma^0 \rightarrow (\Lambda e^+ e^- + \text{arbitrary many photons})$  relative to QED LO calculation of  $\Sigma^0 \rightarrow \Lambda e^+ e^-$

In particular

- lepton bremsstrahlung **beyond** the soft-photon approximation
- explicit calculation of two-photon-exchange ( $1\gamma$ IR) contrb. and correction to  $\Sigma^0 \Lambda \gamma$  vertex
- radiative corrections that involve other hadronic form factors can be safely neglected  
 $\leftrightarrow$  model-independent results in terms of a **single** hadronic parameter  $a$

Precise and **conservative** predictions for  $\mathcal{B}(\Sigma^0 \rightarrow \Lambda \gamma)$  and  $\mathcal{B}(\Sigma^0 \rightarrow \Lambda e^+ e^-)$

Estimate on the correction to the magnetic form-factor slope:  $\Delta a \approx -3.5\%$

*TH and S. Leupold, EPJC 80 (2020), arXiv:1911.02571*

Thank you for listening!

