Radiative corrections in Dalitz decays of π^0, η and η' mesons

Tomáš Husek
IFIC, Universitat de València – CSIC

In collaboration with
K. Kampf, J. Novotný (*Charles U.*), S. Leupold (*Uppsala U.*), E. Goudzovski (*U. Birmingham*)

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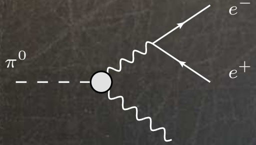
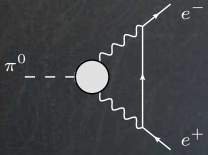
Rare decay of π^0

Introduction

Decay modes of neutral pion:

Process	Branching ratio
$\pi^0 \to \gamma\gamma$	$(98.823 \pm 0.034)\,\%$
$\pi^0 \to e^+e^-\gamma$	$(1.174 \pm 0.035)\,\%$
$\pi^0 \to e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \to e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$





Rare decay $\pi^0 \to e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to *Drell*, NC (1959)
- suppressed in comparison to the decay $\pi^0 \to \gamma\gamma$ by a factor of $2(\alpha m_e/M_\pi)^2$

  $\hookrightarrow$ one-loop structure + helicity suppression
  $\hookrightarrow$ may be sensitive to possible effects of new physics

Rare decay of π^0

KTeV measurement

KTeV-E799-II experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))
$\hookrightarrow$ precise measurements of branching ratio $\pi^0 \to e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma),\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B_{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
- extrapolate full radiative tail beyond $x > 0.95$ (*Bergström*, Z.Ph.C 20 (1983))
- scale the result back by the overall radiative corrections

$\hookrightarrow$ final result for lowest order (no final state radiation)

$$B_{\text{KTeV}}^{\text{no-rad}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov*, PRD 75 (2007))

$$B_{\text{SM}}^{\text{no-rad}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

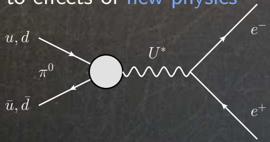
$\hookrightarrow$ interpreted as $3.3\,\sigma$ discrepancy between theory and experiment

Rare decay of π^0

New physics?

- very fashionable to ascribe eventual discrepancies to effects of new physics

BUT



- first, look for more conventional solution (i.e. within SM)
  - ↪ radiative corrections (usually very important)
  - ↪ transition-form-factor modeling: *TH and Leupold,* EPJC 75 (2015)

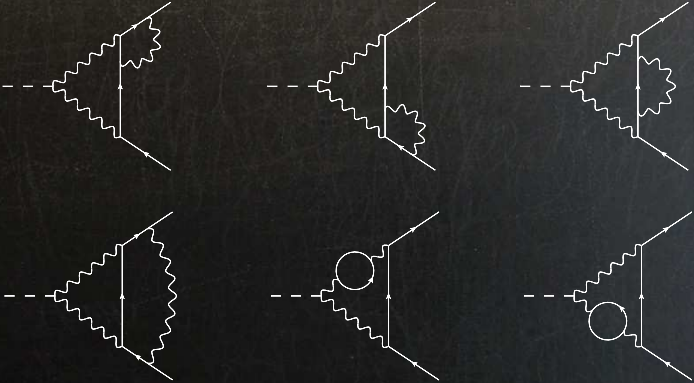
Two-hadron saturation ("LMD+V+P")

$$\mathcal{F}^{\text{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right]$$
$$\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

Radiative corrections to $\pi^0 \to e^+e^-$

Two-loop virtual radiative corrections

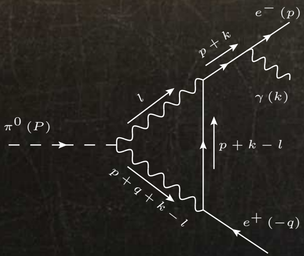
- calculated by *Vaško and Novotný*, JHEP 1110 (2011)

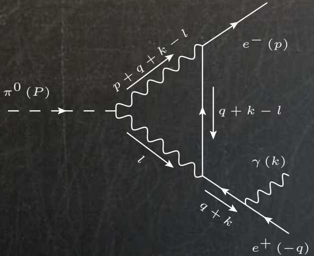


Radiative corrections to $\pi^0 \to e^+e^-$

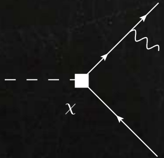
Bremsstrahlung: photon emission from the outer fermion line

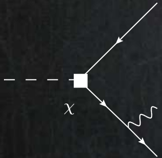
- compensation of IR divergence in 2-loop contributions
  $\hookrightarrow$ *TH, Kampf and Novotný,* EPJC 74 (2014)





- contain UV subdivergences → counter-term tree diagrams with couplig χ





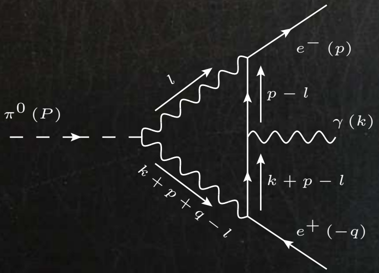
Radiative corrections to $\pi^0 \to e^+e^-$

Bremsstrahlung: photon emission from the inner fermion line (propagator)

Do not forget the third, box diagram, necessary to satisfy the Ward identities

$$\mathcal{M}_{(\lambda)} = \varepsilon^{*\rho}_{(\lambda)}(k)\mathcal{M}^{\mathsf{BS}}_{\rho} \quad \longrightarrow \quad k^{\rho}\mathcal{M}^{\mathsf{BS}}_{\rho} = 0$$

- finite contribution to bremsstrahlung amplitude



Radiative corrections to $\pi^0 \to e^+e^-$

Final results

Size of the radiative corrections (newly calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$
Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$

- original KTeV vs. SM discrepancy reduced to the $2\,\sigma$ level or less
- contact interaction coupling finite part set to

$$\chi^{(\text{r})}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$$

Dalitz decay of the neutral pion

Introduction

Quantity really measured by KTeV

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma)\,,\ x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma(\gamma)\,,\ x > 0.2319)}\bigg|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

↪ Dalitz decay comes into play

- second most important decay channel of the neutral pion
  ↪ branching ratio $(1.174 \pm 0.035)\,\%$

- first studied by *Richard H. Dalitz*, PPSA 64 (1951), whose name it carries

- experimental data of this process provide the information about singly-virtual pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$
  ↪ in particular about its slope parameter a_π

Radiative corrections to $\pi^0 \to e^+e^-\gamma$

Introduction

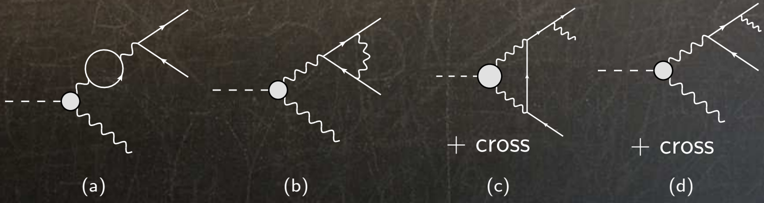
- radiative corrections to the total decay rate of the Dalitz decay
  $\hookrightarrow$ first addressed (numerically) by *Joseph*, NC 16 (1960)

- pioneering study of corrections to the differential decay rate
  $\hookrightarrow$ *Lautrup and Smith*, PRD 3 (1971)
  $\hookrightarrow$ soft-photon approximation

- extended by *Mikaelian and Smith*, PRD 5 (1972)
  $\hookrightarrow$ hard-photon corrections
  $\hookrightarrow$ whole range of bremsstrahlung photon energy
  $\hookrightarrow$ table of values

Radiative corrections to $\pi^0 \to e^+e^-\gamma$

Novel approach



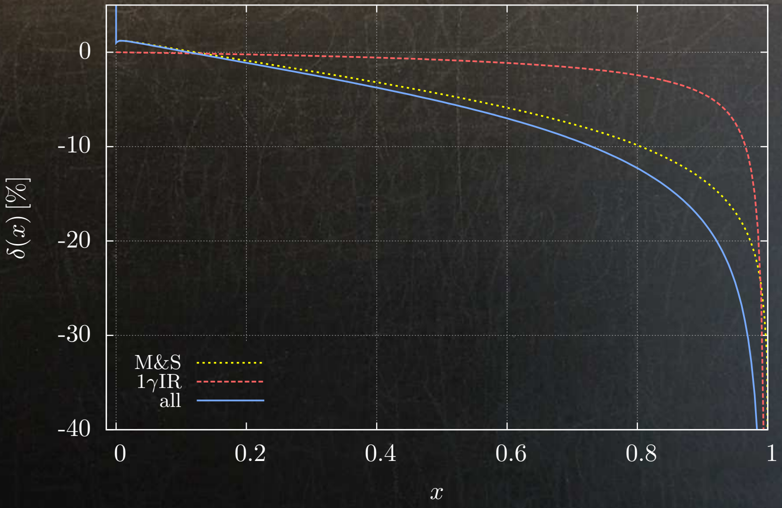
- new calculations motivated by needs of NA48/NA62 experiments at CERN
  $\hookrightarrow$ measure the slope of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$: *Lazzeroni et al.*, PLB 768 (2017)

$$a_\pi^{\text{NA62}} = 3.68(57)\,\%$$

- unlike before no approximation was used
  $\hookrightarrow$ can be used also for related decays $\eta \to \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
  $\hookrightarrow$ propagated into MC generator of NA62 experiment
- *TH, Kampf and Novotný*, PRD 92 (2015)

Radiative corrections to $\pi^0 \to e^+e^-\gamma$

Results



Radiative corrections to $\pi^0 \to e^+e^-\gamma$

Determination of ratio R

Precise and reliable determination of $R \equiv \frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$

↪ for small slope and up to NLO radiative corrections

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x,y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] \mathrm{d}x\,\mathrm{d}y$$

Conservative estimate for uncertainty (a_π, NNLO): $R = 1.1978(6)\,\%$

↪ chosen $a_\pi = 3.55(70)\,\%$, covers

$$a_\pi^{\text{VMD}} = 3.0\,\%\,, \quad a_\pi^{\text{PDG}} = 3.35(31)\,\%\,, \quad a_\pi^{\text{NA62}} = 3.68(57)\,\%$$

Constraint: $1 \simeq \mathcal{B}(\pi^0 \to \gamma\gamma) + \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \to e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \to \gamma\gamma) = 98.8131(6)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) = 1.1836(6)\,\%$$

TH, Goudzovski and Kampf, arXiv:1809.01153

PDG

$$R = 1.188(35)\,\%\,, \quad \mathcal{B}(\pi^0 \to \gamma\gamma) = 98.823(34)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma) = 1.174(35)\,\%$$

Dalitz decays of $\eta^{(\prime)}$

Introduction

$\eta^{(\prime)}$ Dalitz decays

- small branching ratios
  $\hookrightarrow$ hadronic decay modes are open

- access to electromagnetic transition form factors
  $\hookrightarrow$ $\eta^{(\prime)}$-meson structure
  $\hookrightarrow$ valuable input for other quantities and e.g. $g-2$ of a muon
  $\hookrightarrow$ radiative corrections crucial to extract relevant information from data

naive rad. corrections for $\eta \to e^+e^-\gamma$: *Mikaelian and Smith*, PRD 5 2890 (1972)

- numerical values correspond to simple change $M_{\pi^0} \to M_\eta$
  $\hookrightarrow$ π^0 case: *Mikaelian and Smith*, PRD 5 1763 (1972)

- fully inclusive radiative corrections
  $\hookrightarrow$ no momentum or angular cuts on the bremsstrahlung photon applied

Radiative corrections to $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

What should be taken into account?

The $\eta^{(\prime)}$ case compared to π^0

- larger rest mass

$\hookrightarrow$ M_η above muon-pair threshold: $M_\eta > 2m_\mu$
$\hookrightarrow$ $M_{\eta'}$ above lowest-lying resonances: $M_{\eta'} > M_\rho, M_\omega$
  $\hookrightarrow$ sensitive to the widths of resonances
  $\hookrightarrow$ ω narrow, ρ broad resonance in $\pi\pi$ scattering

- strange-flavor content

$\hookrightarrow$ quark-flavor basis
Feldmann et al., PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2}\left[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d\right], \quad j^{\mathrm{s}} \equiv \frac{i}{\sqrt{2}}\left[\bar{s}\gamma_5 s\right]$$

- η-η' mixing: $\langle 0|j^A|\eta^B\rangle = B_0 F_\pi f_A \delta^{AB}$, $\langle \eta^A|\eta^B\rangle = \delta^{AB}$, $A, B \in \{\ell, \mathrm{s}\}$

$$|\eta\rangle = \cos\phi\,|\eta^\ell\rangle - \sin\phi\,|\eta^{\mathrm{s}}\rangle$$
$$|\eta'\rangle = \sin\phi\,|\eta^\ell\rangle + \cos\phi\,|\eta^{\mathrm{s}}\rangle$$

Radiative corrections to $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Novel approach

Full set of NLO QED radiative corrections:
TH, Kampf, Leupold and Novotný, PRD 97 (2018)

- compared to previous approach:
  - ↪ muon loops + hadronic VP
  - ↪ 1γIR at one-loop level
  - ↪ form-factor effects (also in BS)
  - ↪ higher orders in the final-state-lepton mass not neglected
- general framework: three additional processes
  - ↪ also muon decay modes

η case: most of the ingredients in *TH, Kampf and Novotný*, PRD 92 (2015)

η' case: real challenge

↪ resulting framework also applicable to the π^0 case (numerically compatible)
- ↪ overkill (correction to the correction of order 1 %)

Radiative corrections to $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Virtual corrections

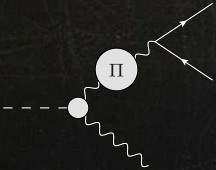
Photon self-energy in the form $\Pi(s) = \Pi_{\mathsf{L}}(s) + \Pi_{\mathsf{H}}(s)$

- lepton loops (electrons and as well muons)

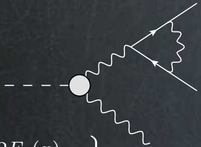
$$\Pi_{\mathsf{L}}(M_P^2 x) = \frac{\alpha}{\pi} \sum_{\ell'=e,\mu} \left\{ \frac{8}{9} - \frac{\beta_{\ell'}^2}{3} + \left(1 - \frac{\beta_{\ell'}^2}{3}\right) \frac{\beta_{\ell'}}{2} \log\left[-\gamma_{\ell'} + i\epsilon\right] \right\}$$

- hadronic contribution

$\hookrightarrow$ *Jegerlehner*, Z.Ph.C 32 (1986)



$$\Pi_{\mathsf{H}}(s) = -\frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_{\mathsf{H}}(s')\,\mathrm{d}s'}{s - s' + i\epsilon}$$



$$\delta^{\mathsf{virt}}(x,y) = \frac{1}{|1+\Pi(M_P^2 x)|^2} - 1 + 2\,\mathrm{Re}\left\{ F_1(x) + \frac{2F_2(x)}{1 + y^2 + \frac{\nu^2}{x}} \right\}$$

Radiative corrections to $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

1γIR contribution at one-loop level

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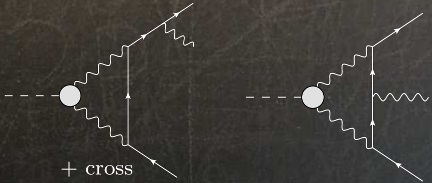


1γIR contribution at one-loop level

- beyond effective approach
- we don't expect substantial model dependence ↪ VMD-inspired model
  ↪ strange-flavor content and η-η' mixing

$$e^2\mathcal{F}^{\mathrm{VMD}}_{\eta\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{8\pi^2F_\pi}\frac{2e^2}{3}$$
$$\times\left[\frac{5}{3}\frac{\cos\phi}{f_\ell}\frac{M^4_{\omega/\rho}}{(p^2-M^2_{\omega/\rho})(q^2-M^2_{\omega/\rho})}-\frac{\sqrt{2}}{3}\frac{\sin\phi}{f_s}\frac{M^4_\phi}{(p^2-M^2_\phi)(q^2-M^2_\phi)}\right]$$

Radiative corrections to $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Bremsstrahlung

+ cross

- slope not negligible
- for η: expansion in slope a would be still (somewhat) suitable

$$\mathcal{F}\big((p_\gamma + p_{e^+} + p_{e^-})^2\big) \simeq \mathcal{F}(M_P^2 x)\left[1 + a\,\frac{2p_\gamma \cdot (p_{e^+} + p_{e^-})}{M_P^2}\right]$$

- for η': such an expansion not applicable anymore
  $\hookrightarrow$ BS necessarily depends on the form-factor model

sensitivity to width of ρ meson $\hookrightarrow$ recent dispersive calculations used
Hanhart et al., EPJC 73 (2013), EPJC 77 (2017)

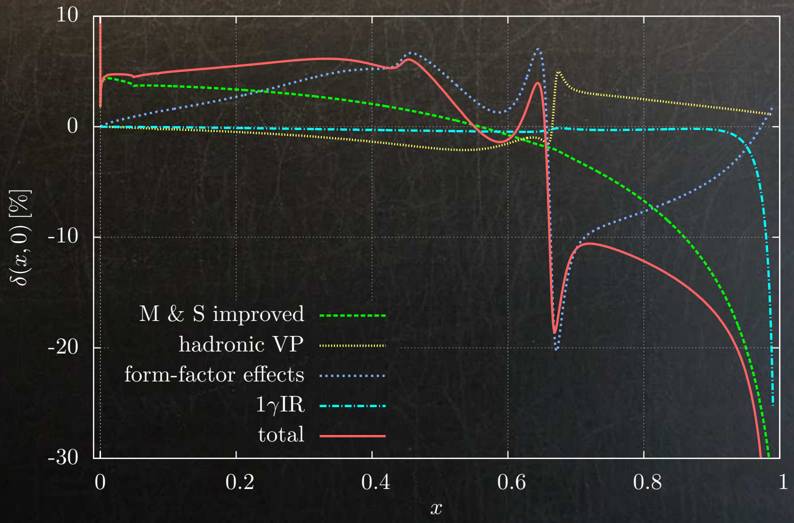
Källén–Lehmann spectral representation $\to$ common spectral density function

$$\frac{\mathcal{F}(q^2)}{\mathcal{F}(0)} \simeq 1 + q^2 \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathcal{A}(s)\,\mathrm{d}s}{q^2 - s + i\epsilon}$$

$$\mathcal{A}(s) = w_\omega \mathcal{A}_\omega(s) + w_\phi \mathcal{A}_\phi(s) - \frac{\kappa}{96\pi^2 F_\pi^2}\left[1 - \frac{4m_\pi^2}{s}\right]^{3/2} P(s)\,R(s)\,|\Omega(s)|^2$$

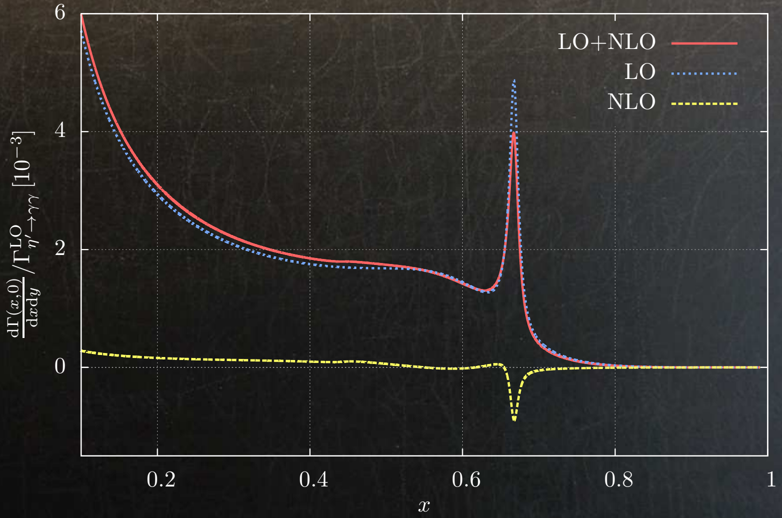
Radiative corrections to $\eta' \to e^+e^-\gamma$ decays

The overall NLO correction $\delta(x,0)$ in comparison to its constituents



Radiative corrections to $\eta' \to e^+e^-\gamma$ decays

The two-fold differential decay width $\mathrm{d}\Gamma(x,0)$ at NLO



Summary

All NLO QED radiative corrections for discussed processes are now available
↪ can be taken into account in future experimental analyses

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
  ↪ THS model: *TH and S. Leupold,* EPJC 75 (2015)
- $\pi^0 \to e^+e^-\gamma$
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  ↪ precise determination of R: *TH, Goudzovski and Kampf,* arXiv:1809.01153
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Ancillary files available together with the papers

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Thank you for listening!