

Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta$  ( $\eta'$ ) Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

Radiative corrections in Dalitz decays of  $\pi^0$ ,  $\eta$  and  $\eta'$  mesons

**Tomáš Husek**

*IFIC, Universitat de València – CSIC*

In collaboration with

**K. Kampf, J. Novotný (Charles U.), S. Leupold (Uppsala U.), E. Goudzovski (U. Birmingham)**

Durham, NC

September 19, 2018



Contents

Introduction

$\pi^0$  rare decay

- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results

$\pi^0$  Dalitz decay

- Introduction
- Radiative corrections
- Results

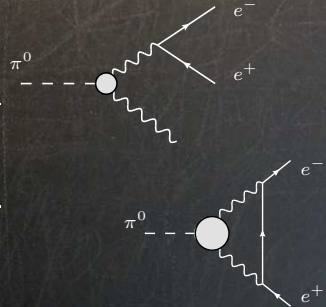
$\eta$  ( $\eta'$ ) Dalitz decays

- Introduction
- Radiative corrections
- Specific contributions
- Results

Summary

## Decay modes of neutral pion:

Process	Branching ratio
$\pi^0 \rightarrow \gamma\gamma$	$(98.823 \pm 0.034) \%$
$\pi^0 \rightarrow e^+e^-\gamma$	$(1.174 \pm 0.035) \%$
$\pi^0 \rightarrow e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \rightarrow e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



## Rare decay $\pi^0 \rightarrow e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to **Drell, NC (1959)**
- suppressed in comparison to the decay  $\pi^0 \rightarrow \gamma\gamma$  by a factor of  $2(\alpha m_e/M\pi)^2$ 
  - $\hookrightarrow$  one-loop structure + helicity suppression
  - $\hookrightarrow$  may be sensitive to possible effects of new physics

Contents

Introduction

$\pi^0$  rare decay

Introduction

Radiative corrections

Virtual corrections

Bremsstrahlung

Results

$\pi^0$  Dalitz decay

Introduction

Radiative corrections

Results

$\eta^{(\prime)}$  Dalitz decays

Introduction

Radiative corrections

Specific contributions

Results

Summary

KTeV-E799-II experiment at Fermilab (*Abouzaid et al., PRD 75 (2007)*)  
 $\hookrightarrow$  **precise** measurements of branching ratio  $\pi^0 \rightarrow e^+e^-$  (794 candidates)

$$\frac{\Gamma(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of  $x$

$$B_{\text{KTeV}}(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value  $(6.46 \pm 0.33) \times 10^{-8}$  mainly based on this result
- extrapolate full radiative tail beyond  $x > 0.95$  (*Bergström, Z.Ph.C 20 (1983)*)
- scale the result back by the overall radiative corrections

$\hookrightarrow$  **final result** for lowest order (no final state radiation)

$$B_{\text{KTeV}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov, PRD 75 (2007)*)

$$B_{\text{SM}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

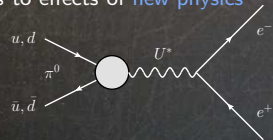
$\hookrightarrow$  interpreted as **3.3  $\sigma$  discrepancy** between theory and experiment

Contents

- Introduction
- $\pi^0$  rare decay
- Introduction**
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta$  ( $\eta'$ ) Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

- very fashionable to ascribe eventual discrepancies to effects of new physics

**BUT**



- first, look for more conventional solution (i.e. within SM)
  - ↔ radiative corrections (usually very important)
  - ↔ transition-form-factor modeling: *TH and Leupold, EPJC 75 (2015)*

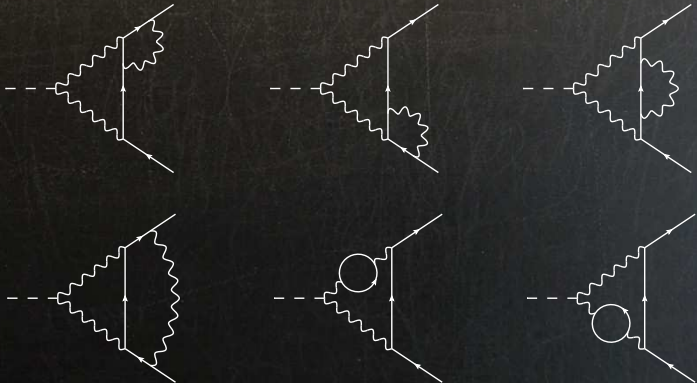
Two-hadron saturation (“LMD+V+P”)

$$\mathcal{F}_{\pi^0 \gamma^* \gamma^*}^{\text{THS}}(p^2, q^2) = -\frac{N_c}{12\pi^2 F} \left[ \frac{M_{V_1}^4 M_{V_2}^4}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)(q^2 - M_{V_1}^2)(q^2 - M_{V_2}^2)} \right] \times \left\{ 1 + \frac{\kappa}{2N_c} \frac{p^2 q^2}{(4\pi F)^4} - \frac{4\pi^2 F^2 (p^2 + q^2)}{N_c M_{V_1}^2 M_{V_2}^2} \left[ 6 + \frac{p^2 q^2}{M_{V_1}^2 M_{V_2}^2} \right] \right\}$$

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections**
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

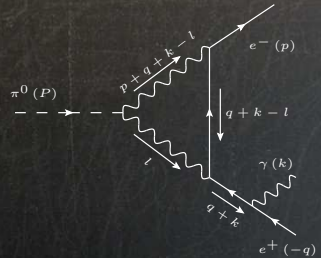
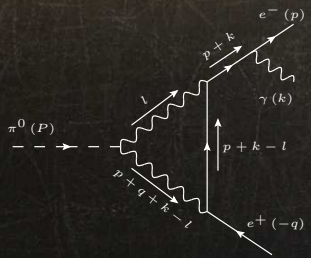
- calculated by *Vaško and Novotný, JHEP 1110 (2011)*



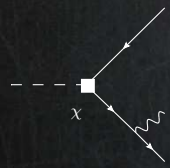
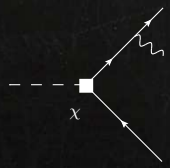
**Contents**

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung**
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

- compensation of **IR** divergence in 2-loop contributions  
 $\hookrightarrow$  *TH, Kampf and Novotný, EPJC 74 (2014)*



- contain **UV** subdivergences  $\rightarrow$  counter-term tree diagrams with couplig  $\chi$



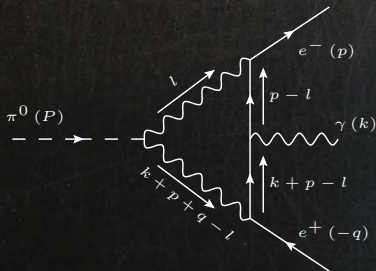
Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung**
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

Do not forget the third, **box** diagram, necessary to satisfy the **Ward identities**

$$\mathcal{M}_{(\lambda)} = \varepsilon_{(\lambda)}^{*\rho}(k) \mathcal{M}_{\rho}^{\text{BS}} \longrightarrow k^{\rho} \mathcal{M}_{\rho}^{\text{BS}} = 0$$

- **finite** contribution to bremsstrahlung amplitude



Contents

- Introduction
- $\pi^0$  rare decay
  - Introduction
  - Radiative corrections
  - Virtual corrections
  - Bremsstrahlung
  - Results
- $\pi^0$  Dalitz decay
  - Introduction
  - Radiative corrections
  - Results
- $\eta^{(\prime)}$  Dalitz decays
  - Introduction
  - Radiative corrections
  - Specific contributions
  - Results
- Summary

Size of the radiative corrections (**newly** calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2) \%$$

- can be thought as model-independent
- differs **significantly** from previous **approximate** calculations

*Bergström, Z.Ph.C 20 (1983):  $\delta(0.95) = -13.8 \%$*

*Dorokhov et al., EPJC 55 (2008):  $\delta(0.95) = -13.3 \%$*

- original KTeV vs. SM discrepancy reduced to the  $2\sigma$  level or less
- contact interaction coupling finite part set to

$$\chi_{\text{LMD}}^{(r)}(M_\rho) = 2.2 \pm 0.9$$



## Contents

Introduction

$\pi^0$  rare decay

Introduction

Radiative corrections

Virtual corrections

Bremsstrahlung

Results

$\pi^0$  Dalitz decay

Introduction

Radiative corrections

Results

$\eta(\prime)$  Dalitz decays

Introduction

Radiative corrections

Specific contributions

Results

Summary

Quantity **really** measured by KTeV

$$\left. \frac{\Gamma(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma(\gamma), x > 0.2319)} \right|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

$\hookrightarrow$  Dalitz decay comes into play

- **second** most important decay channel of the neutral pion  
 $\hookrightarrow$  branching ratio  $(1.174 \pm 0.035) \%$
- first studied by **Richard H. Dalitz**, PPSA 64 (1951), whose name it carries
- experimental data of this process provide the information about **singly-virtual pion transition form factor**  $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$   
 $\hookrightarrow$  in particular about its **slope** parameter  $a_\pi$

Contents

Introduction

$\pi^0$  rare decay

Introduction

Radiative corrections

Virtual corrections

Bremsstrahlung

Results

$\pi^0$  Dalitz decay

Introduction

Radiative corrections

Results

$\eta(\prime)$  Dalitz decays

Introduction

Radiative corrections

Specific contributions

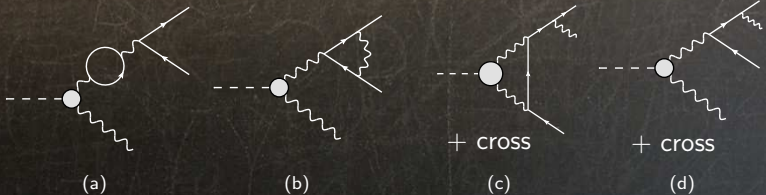
Results

Summary

- radiative corrections to the **total** decay rate of the Dalitz decay  
 $\hookrightarrow$  first addressed (numerically) by *Joseph, NC 16 (1960)*
- pioneering study of corrections to the **differential** decay rate  
 $\hookrightarrow$  *Lautrup and Smith, PRD 3 (1971)*  
 $\hookrightarrow$  soft-photon approximation
- extended by *Mikaelian and Smith, PRD 5 (1972)*  
 $\hookrightarrow$  hard-photon corrections  
 $\hookrightarrow$  **whole** range of bremsstrahlung photon energy  
 $\hookrightarrow$  table of values

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary



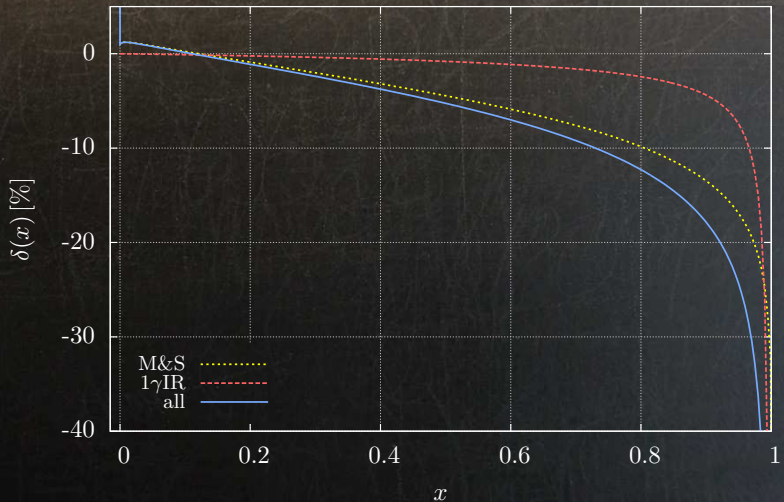
- new calculations motivated by needs of NA48/NA62 experiments at CERN
  - ↪ measure the slope of  $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$ : *Lazzeroni et al., PLB 768 (2017)*

$$a_{\pi}^{\text{NA62}} = 3.68(57) \%$$

- unlike before **no approximation** was used
  - ↪ can be used also for related decays  $\eta \rightarrow \ell^+\ell^-\gamma$  etc.
- C++ code returns the correction for any given  $x$  and  $y$ 
  - ↪ propagated into **MC generator** of NA62 experiment
- *TH, Kampf and Novotný, PRD 92 (2015)*

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results**
- $\eta^{(\prime)}$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary



## Contents

Introduction

$\pi^0$  rare decay

Introduction  
Radiative  
corrections

Virtual  
corrections

Brems-  
strahlung  
Results

$\pi^0$  Dalitz  
decay

Introduction  
Radiative  
corrections

**Results**

$\eta(\prime)$  Dalitz  
decays

Introduction  
Radiative  
corrections

Specific  
contributions  
Results

Summary

Precise and reliable determination of  $R \equiv \frac{\Gamma(\pi^0 \rightarrow e^+e^-\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)}$

$\hookrightarrow$  for small slope and up to NLO radiative corrections

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x, y)) \frac{(1-x)^3}{4x} \left[ 1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] dx dy$$

Conservative estimate for uncertainty ( $a_\pi$ , NNLO):  $R = 1.1978(6) \%$

$\hookrightarrow$  chosen  $a_\pi = 3.55(70) \%$ , covers

$$a_\pi^{\text{VMD}} = 3.0 \%, \quad a_\pi^{\text{PDG}} = 3.35(31) \%, \quad a_\pi^{\text{NA62}} = 3.68(57) \%$$

Constraint:  $1 \simeq \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) + \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \rightarrow e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.8131(6) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) = 1.1836(6) \%$$

*TH, Goudzovski and Kampf, arXiv:1809.01153*

**PDG**

$$R = 1.188(35) \%, \quad \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.823(34) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma) = 1.174(35) \%$$

## Contents

Introduction

$\pi^0$  rare decay

Introduction

Radiative  
corrections

Virtual  
corrections

Brems-  
strahlung

Results

$\pi^0$  Dalitz  
decay

Introduction

Radiative  
corrections

Results

$\eta^{(\prime)}$  Dalitz  
decays

Introduction

Radiative  
corrections

Specific  
contributions

Results

Summary

## $\eta^{(\prime)}$ Dalitz decays

- small branching ratios  
 $\hookrightarrow$  hadronic decay modes are open
- access to electromagnetic transition form factors  
 $\hookrightarrow \eta^{(\prime)}$ -meson structure  
 $\hookrightarrow$  valuable input for other quantities and e.g.  $g - 2$  of a muon  
 $\hookrightarrow$  radiative corrections crucial to **extract** relevant information from data

**naive** rad. corrections for  $\eta \rightarrow e^+e^-\gamma$ : *Mikaelian and Smith, PRD 5 2890 (1972)*

- numerical values correspond to simple change  $M_{\pi^0} \rightarrow M_\eta$   
 $\hookrightarrow \pi^0$  case: *Mikaelian and Smith, PRD 5 1763 (1972)*
- fully inclusive radiative corrections  
 $\hookrightarrow$  **no** momentum or angular cuts on the bremsstrahlung photon applied

## Contents

Introduction

$\pi^0$  rare decay

Introduction

Radiative corrections

Virtual corrections

Bremsstrahlung

Results

$\pi^0$  Dalitz decay

Introduction

Radiative corrections

Results

$\eta^{(\prime)}$  Dalitz decays

Introduction

Radiative corrections

Specific contributions

Results

Summary

## The $\eta^{(\prime)}$ case compared to $\pi^0$

- larger rest mass

↪  $M_\eta$  above muon-pair threshold:  $M_\eta > 2m_\mu$

↪  $M_{\eta'}$  above lowest-lying resonances:  $M_{\eta'} > M_\rho, M_\omega$

↪ sensitive to the **widths** of resonances

↪  $\omega$  narrow,  $\rho$  **broad** resonance in  $\pi\pi$  scattering

- **strange-flavor content**

↪ quark-flavor basis

*Feldmann et al.*, PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2} [\bar{u}\gamma_5 u + \bar{d}\gamma_5 d], \quad j^s \equiv \frac{i}{\sqrt{2}} [\bar{s}\gamma_5 s]$$

-  $\eta$ - $\eta'$  **mixing**:  $\langle 0 | j^A | \eta^B \rangle = B_0 F_\pi f_A \delta^{AB}$ ,  $\langle \eta^A | \eta^B \rangle = \delta^{AB}$ ,  $A, B \in \{\ell, s\}$

$$|\eta\rangle = \cos\phi |\eta^\ell\rangle - \sin\phi |\eta^s\rangle$$

$$|\eta'\rangle = \sin\phi |\eta^\ell\rangle + \cos\phi |\eta^s\rangle$$

## Contents

### Introduction

#### $\pi^0$ rare decay

#### Introduction

#### Radiative corrections

#### Virtual corrections

#### Bremsstrahlung

#### Results

#### $\pi^0$ Dalitz decay

#### Introduction

#### Radiative corrections

#### Results

#### $\eta^{(\prime)}$ Dalitz decays

#### Introduction

#### Radiative corrections

#### Specific contributions

#### Results

#### Summary

Full set of NLO QED radiative corrections:

*TH, Kampf, Leupold and Novotný, PRD 97 (2018)*

- compared to previous approach:
  - ↪ muon loops + **hadronic** VP
  - ↪ **1 $\gamma$ IR** at one-loop level
  - ↪ **form-factor** effects (also in BS)
  - ↪ higher orders in the final-state-lepton mass **not** neglected

- general framework: **three** additional processes
  - ↪ also muon decay modes

$\eta$  case: **most** of the ingredients in *TH, Kampf and Novotný, PRD 92 (2015)*

$\eta'$  case: real challenge

- ↪ resulting framework also **applicable** to the  $\pi^0$  case (numerically compatible)
  - ↪ overkill (correction to the correction of order 1%)



## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta^{(\prime)}$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions**
- Results
- Summary

Photon self-energy in the form  $\Pi(s) = \Pi_L(s) + \Pi_H(s)$

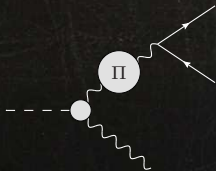
- lepton loops (electrons and as well **muons**)

$$\Pi_L(M_P^2 x) = \frac{\alpha}{\pi} \sum_{\ell'=e,\mu} \left\{ \frac{8}{9} - \frac{\beta_{\ell'}^2}{3} + \left( 1 - \frac{\beta_{\ell'}^2}{3} \right) \frac{\beta_{\ell'}}{2} \log [-\gamma_{\ell'} + i\epsilon] \right\}$$

- **hadronic** contribution

↪ *Jegerlehner, Z.Ph.C 32 (1986)*

$$\Pi_H(s) = -\frac{s}{4\pi^2 \alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_H(s') ds'}{s - s' + i\epsilon}$$



$$\delta^{\text{virt}}(x, y) = \frac{1}{|1 + \Pi(M_P^2 x)|^2} - 1 + 2 \text{Re} \left\{ F_1(x) + \frac{2F_2(x)}{1 + y^2 + \frac{\nu^2}{x}} \right\}$$

## Contents

Introduction

$\pi^0$  rare decay

Introduction  
Radiative corrections

Virtual corrections  
Bremsstrahlung  
Results

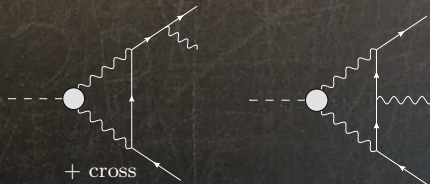
$\pi^0$  Dalitz decay

Introduction  
Radiative corrections  
Results

$\eta^{(\prime)}$  Dalitz decays

Introduction  
Radiative corrections  
**Specific contributions**  
Results

Summary



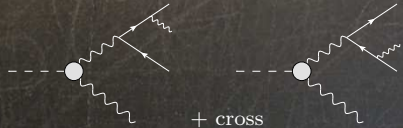
## 1 $\gamma$ IR contribution at one-loop level

- beyond effective approach
- we don't expect substantial model dependence  $\leftrightarrow$  VMD-inspired model  $\leftrightarrow$  strange-flavor content and  $\eta$ - $\eta'$  mixing

$$e^2 \mathcal{F}_{\eta\gamma^*\gamma^*}^{\text{VMD}}(p^2, q^2) = -\frac{N_c}{8\pi^2 F_\pi} \frac{2e^2}{3} \times \left[ \frac{5 \cos \phi}{3} \frac{f_\ell}{f_l} \frac{M_{\omega/\rho}^4}{(p^2 - M_{\omega/\rho}^2)(q^2 - M_{\omega/\rho}^2)} - \frac{\sqrt{2} \sin \phi}{3} \frac{f_s}{f_s} \frac{M_\phi^4}{(p^2 - M_\phi^2)(q^2 - M_\phi^2)} \right]$$

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta^{(\prime)}$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions**
- Results
- Summary



- slope not negligible
- for  $\eta$ : expansion in slope  $a$  would be **still** (somewhat) suitable

$$\mathcal{F}((p_\gamma + p_{e^+} + p_{e^-})^2) \simeq \mathcal{F}(M_P^2 x) \left[ 1 + a \frac{2p_\gamma \cdot (p_{e^+} + p_{e^-})}{M_P^2} \right]$$

- for  $\eta'$ : such an expansion **not applicable** anymore
- $\hookrightarrow$  BS necessarily depends on the form-factor model

sensitivity to width of  $\rho$  meson  $\hookrightarrow$  recent **dispersive** calculations used  
*Hanhart et al., EPJC 73 (2013), EPJC 77 (2017)*

Källén–Lehmann spectral representation  $\rightarrow$  common spectral density function

$$\frac{\mathcal{F}(q^2)}{\mathcal{F}(0)} \simeq 1 + q^2 \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathcal{A}(s) ds}{q^2 - s + i\epsilon}$$

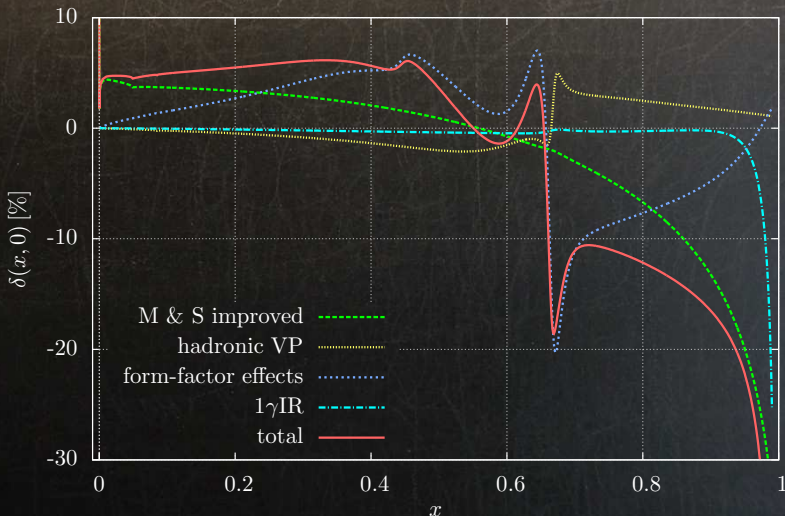
$$\mathcal{A}(s) = w_\omega \mathcal{A}_\omega(s) + w_\phi \mathcal{A}_\phi(s) - \frac{\kappa}{96\pi^2 F_\pi^2} \left[ 1 - \frac{4m_\pi^2}{s} \right]^{3/2} P(s) R(s) |\Omega(s)|^2$$

# Radiative corrections to $\eta' \rightarrow e^+e^-\gamma$ decays

The overall NLO correction  $\delta(x, 0)$  in comparison to its constituents

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta'(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results
- Summary

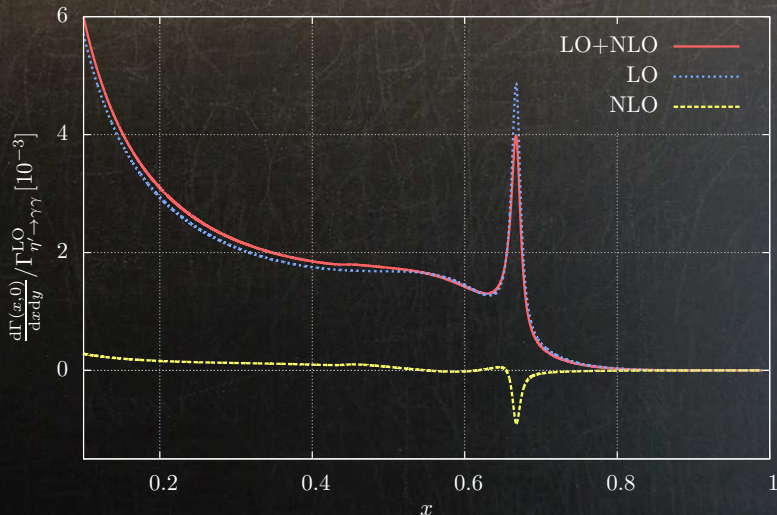


# Radiative corrections to $\eta' \rightarrow e^+e^-\gamma$ decays

The two-fold differential decay width  $d\Gamma(x,0)$  at NLO

## Contents

- Introduction
- $\pi^0$  rare decay
- Introduction
- Radiative corrections
- Virtual corrections
- Bremsstrahlung
- Results
- $\pi^0$  Dalitz decay
- Introduction
- Radiative corrections
- Results
- $\eta(\prime)$  Dalitz decays
- Introduction
- Radiative corrections
- Specific contributions
- Results**
- Summary



## Contents

- Introduction
- $\pi^0$  rare decay
  - Introduction
  - Radiative corrections
  - Virtual corrections
  - Bremsstrahlung
  - Results
- $\pi^0$  Dalitz decay
  - Introduction
  - Radiative corrections
  - Results
- $\eta^{(\prime)}$  Dalitz decays
  - Introduction
  - Radiative corrections
  - Specific contributions
  - Results
- Summary

All NLO QED radiative corrections for discussed processes are now available

↔ can be taken into account in **future** experimental analyses

-  $\pi^0 \rightarrow e^+e^-$

*Vaško and Novotný, JHEP 1110 (2011)*

*TH, Kampf and Novotný, EPJC 74 (2014)*

↔ THS model: *TH and Leupold, EPJC 75 (2015)*

-  $\pi^0 \rightarrow e^+e^-\gamma$

*TH, Kampf and Novotný, PRD 92 (2015)*

↔ **precise** determination of  $R$ : *TH, Goudzovski and Kampf, arXiv:1809.01153*

-  $\eta^{(\prime)} \rightarrow \ell^+\ell^-\gamma$

*TH, Kampf, Leupold and Novotný, PRD 97 (2018)*

Ancillary files available together with the papers

Contents

- Introduction
- $\pi^0$  rare decay
  - Introduction
  - Radiative corrections
    - Virtual corrections
    - Bremsstrahlung
    - Results
- $\pi^0$  Dalitz decay
  - Introduction
  - Radiative corrections
    - Results
- $\eta^{(\prime)}$  Dalitz decays
  - Introduction
  - Radiative corrections
    - Specific contributions
    - Results
- Summary

All NLO QED radiative corrections for discussed processes are now available

↔ can be taken into account in **future** experimental analyses

-  $\pi^0 \rightarrow e^+e^-$

*Vaško and Novotný, JHEP 1110 (2011)*

*TH, Kampf and Novotný, EPJC 74 (2014)*

↔ THS model: *TH and Leupold, EPJC 75 (2015)*

-  $\pi^0 \rightarrow e^+e^-\gamma$

*TH, Kampf and Novotný, PRD 92 (2015)*

↔ **precise** determination of  $R$ : *TH, Goudzovski and Kampf, arXiv:1809.01153*

-  $\eta^{(\prime)} \rightarrow \ell^+\ell^-\gamma$

*TH, Kampf, Leupold and Novotný, PRD 97 (2018)*

Ancillary files available together with the papers

Thank you for listening!