Dipole response in $^{132}\text{Sn}$ within a self-consistent multiphonon approach

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A self-consistent calculation is performed within an equation of motion method to study the electric dipole response in the neutron rich $^{132}\text{Sn}$ with special attention at the low-energy excitations associated with the pygmy dipole resonance. A basis of one- plus two-phonon states is constructed by solving iteratively a set of equations of motion and adopted to bring the nuclear Hamiltonian to diagonal form. The phonons are generated in a Tamm-Dancoff approximation using a Hartree-Fock basis. The Hamiltonian is composed of an intrinsic kinetic term and an optimized two-body chiral potential. If complemented with a weak phenomenological density dependent term, such a potential improves substantially the description of the dipole response as compared to other realistic interactions. The two-phonon states play an important role in determining the fine structure of the giant dipole resonance and in enhancing the fragmentation of the strength at low energy, in fair agreement with the available data. Insights into the nature of the excitations building up the pygmy resonance are gained from the analysis of the phonon composition and shell structure of the states.


I. INTRODUCTION

Evidence of low-energy electric dipole transitions below the giant dipole resonance (GDR) in nuclei with neutron excess was gained in early experiments [1,2]. These excitations were interpreted soon after [3] as a manifestation of a soft collective mode, dubbed pygmy dipole resonance (PDR), induced by a translational oscillation of the neutron excess against an $N=Z$ core.

The interest toward such a mode increased dramatically with the advent of radioactive beams. Since a relativistic Coulomb excitation experiment had detected an appreciable strength at low energy in neutron rich oxygen isotopes [4], several different techniques were employed successfully in the search of low-lying dipole transitions in stable and unstable neutron rich nuclei. A comprehensive and updated list of references can be found in a recent review [5].

Among them, radioactive beam experiments have extracted an appreciable dipole strength just above the neutron decay threshold in tin isotopes around $^{132}\text{Sn}$ [6] while $(\gamma,\gamma')$ [7–16] complemented with $(\alpha,\alpha'\gamma)$ experiments [17–19] have produced dense discrete spectra of weakly excited levels in neutron rich nuclei over different mass regions. Last but not least, inelastic proton scattering experiments were able to produce the full electric dipole spectrum of $^{208}\text{Pb}$ with great accuracy and, in particular, to extract the low-lying transition strength below and above the neutron decay threshold [20,21].

The structure of these low-energy states was investigated in several theoretical approaches. A fairly exhaustive list of references can be found in Refs. [5,22,23]. Many microscopic calculations were carried out in random-phase approximation (RPA) rooted in energy density functional theories. The list includes Hartree-Fock (HF) plus RPA [24–32] or, for open shell nuclei, Hartree-Fock-Bogoliubov (HFB) plus quasiparticle RPA (QRPA) [33–38]. We may distinguish further between nonrelativistic (Q)RPA using Skyrme [26,33,34] or Gogny forces [24,38,39] and relativistic RPA (RRPA) using density functionals derived from meson-nucleon Lagrangians treated in mean field approximation [25,29,32].

Several extensions of (Q)RPA were also adopted in order to describe properly the fragmentation of the mode and to investigate the possible role played by other low-lying modes in determining the structure of the observed dipole spectra. We mention the QRPA plus phonon coupling [40], the second RPA [41], the quasiparticle-phonon model (QPM) [8,16,42], and the relativistic quasiparticle time-blocking approximation (RTBA) [43–45].

The goal of the theoretical studies was not only the understanding of the nature of the low-lying dipole response. Several papers have linked the observed low-lying strength to the thickness of the neutron skin and the symmetry energy [25,26,29,31,32,46,47] and pointed out the relevance of these quantities to the properties of neutron stars [48,49].

As pointed out already, most of the self-consistent (Q)RPA calculations as well as their extensions were based on nonrelativistic or relativistic energy density functionals or used phenomenological separable interactions as in the case of the QPM [50].

An alternative approach was proposed recently by Roth and co-workers [51]. They performed a HFB plus QRPA calculation using a realistic effective interaction derived directly from bare nucleon-nucleon (NN) potentials and obtained realistic spectra and responses once a corrective phenomenological density dependent contact term simulating a three-body force was added to the two-body potential.

Very recently, we performed analogous self-consistent calculations in quasiparticle Tamm-Dancoff approximation (QTDA) and QRPA [52] using a $V_{\text{low}}$ potential derived from the CD-Bonn NN interaction [53]. We also had to add the same density dependent term used in Ref. [51] in...
order to obtain spectra compatible with the experimental data. The calculations produced for the Sn isotopes dipole strength distributions were in global fair agreement with the data obtained in recent experiments [6]. On the other hand, QTDA and QRPA were far from being able to reproduce the fragmentation of the strength, especially the discrete spectra observed in (γ,γ′) and (α,α′γ) [18].

To enhance the fragmentation of the dipole strength, especially at low energy, we adopt here an equation of motion phonon method (EMPM) [54,55] in its upgraded version [56]. The method constructs and solves iteratively a set of equations of motion to generate a multiphonon basis built of phonons obtained in the Tamm-Dancoff approximation (TDA). In such a multiphonon basis, the Hamiltonian matrix has a simple structure and is easily turned into diagonal form.

The EMPM adopts a Hamiltonian of general type and treats one-phonon as well as multiphonon states on the same footing. It was applied to 208Pb [57] and yielded a weaker coupling constant, in order to get an overlap between the theoretical and experimental GDR bumps. This potential, accounts effectively for the part of short range repulsion [56]. The method constructs and solves iteratively a set of equations of motion to generate a multiphonon basis built of phonons obtained in the Tamm-Dancoff approximation (TDA). In such a multiphonon basis, the Hamiltonian matrix has a simple structure and is easily turned into diagonal form.

II. BRIEF OUTLINE OF THE METHOD

Let us consider the Hamiltonian

\[ H = H_0 + V. \]  

In the \( j - j \) coupled scheme, the one- and two-body pieces assume the second quantized expressions

\[ H_0 = \sum_r [r J/Gamma_1 M/Gamma_1 e_r (a_r^+ \times b_r)^0], \]  

\[ V = -\frac{1}{4} \sum_{rsqt} [\Gamma J/Gamma_1^1/2 V_{rsqt}^\Gamma ((a_r^+ \times a_t)^0 \times (b_q \times b_t)^0)]^0, \]

where

\[ V_{rsqt}^\Gamma = \langle (r \times s)^\Gamma | V | (s \times r)^\Gamma \rangle. \]

Following French notation [64], we have put \( b_r = (-)^{\nu+m_r} a_{j_r-m_r}, |\Gamma| = 2F + 1 \) and

\[ |(r \times s)^\Gamma \rangle = \sum_{m_r m_s} \langle j_r m_r, j_s m_s | M_r M_t | c_r j_r m_r c_t j_s m_s \rangle. \]

where \( c_t \) denote all the additional quantum numbers.

It is useful to write the two-body potential (3) in the recoupled form

\[ V = \frac{1}{4} \sum_{rsqt} [\sigma J/Gamma_1^1/2 F_{rsqt}^\sigma ((a_r^+ \times b_t)^0 \times (a_t^+ \times b_r)^0)]^0, \]

where

\[ F_{rsqt}^\sigma = \sum_{\Gamma} [\Gamma (\gamma^{\sigma - +} - (\gamma^{\sigma - +})^\dagger) W(rsqt; \sigma \Gamma)] V_{rsqt}^\Gamma, \]

and \( W(rsqt; \sigma \Gamma) \) are Racah coefficients.

The primary goal of the method is to generate a basis of \( n \)-phonon states \( |n; \beta \rangle \) of the form

\[ |n; \beta \rangle = \sum_{n, \alpha} C_{n, \alpha}^\beta (O_{1}^\dagger \times |n - 1, \alpha \rangle)^\beta, \]

where the TDA particle-hole (p-h) phonon operator

\[ O_{1}^\dagger = \sum_{ph} \chi_{ph}^\dagger (a_p^+ \times b_h)^\dagger \]

acts on the \((n - 1)\)-phonon states \( |n - 1, \alpha \rangle \).

The procedure goes through several steps. We start with writing the equations of motion

\[ \langle n, \beta | [(H, O_{1}^\dagger)] \times |n - 1, \alpha \rangle \rangle \]

\[ = (E_\beta - E_\alpha) (n, \beta | (O_{1}^\dagger \times |n - 1, \alpha \rangle \rangle \]

Using the Wigner-Eckart theorem we get

\[ \langle n, \beta | [(H, O_{1}^\dagger)] \times |n - 1, \alpha \rangle \]

\[ = (E_\beta - E_\alpha) (n, \beta | O_{1}^\dagger \times (n - 1, \alpha \rangle. \]

We then expand the commutator and invert Eq. (9) in order to express the p-h operators, present in the expanded commutator,
in terms of the phonon operators $O^{1}_{\lambda}$. The outcome of this action is [56]
\begin{equation}
\sum_{\lambda'\gamma} A^\beta(\lambda\alpha,\lambda'\gamma)X^\beta_{\lambda'\gamma} = E_\beta X^\beta_{\lambda\alpha},
\end{equation}

where $X$ defines the amplitude
\begin{equation}
X^\rho_{\lambda\alpha} = \langle n,\rho \parallel O^{1}_{\lambda} || n - 1,\alpha \rangle
\end{equation}
and $A$ is a matrix of the simple structure
\begin{equation}
A^\beta(\lambda\alpha,\lambda'\gamma) = (E_\lambda + E_\alpha)\delta_{\lambda,\lambda'}\delta_{\alpha,\gamma} + \sum_\sigma W(\beta\lambda'\alpha'\sigma,\gamma\lambda)\mathcal{V}^\rho_{\lambda'\alpha'\gamma\lambda'\gamma}.
\end{equation}

Here, the phonon-phonon potential is given by
\begin{equation}
\mathcal{V}^\rho_{\lambda'\alpha'\gamma\lambda'\gamma} = \sum_{rs} V^\rho_{\lambda'\lambda}(rs)\rho^\rho_{\alpha'\alpha}(\langle r \times s \rangle),
\end{equation}

where the labels $(rs)$ run over particle $(rs = pp')$ and hole $(rs = hh')$ states. In the above equation, we have introduced the $n$-phonon density matrix
\begin{equation}
\rho^\rho_{\alpha'\alpha}(\langle r \times s \rangle) = \langle n;\alpha' \mid [a^\dagger_r \times b_s]^{\rho}\mid n;\alpha\rangle
\end{equation}
and the potential
\begin{equation}
V^\rho_{\lambda'\lambda}(rs) = \sum_{tq} \rho_{\lambda\lambda'}(\langle q \times t \rangle)F^\rho_{tqrs},
\end{equation}

where $\rho_{\lambda\lambda'}$ is the TDA matrix density given by
\begin{equation}
\rho_{\lambda\lambda'}(\langle r \times s \rangle) = \langle \lambda'\parallel (a^\dagger_r \times b_s)^\rho \mid \lambda \rangle
= [\lambda \lambda' \sigma]^{1/2} \sum_i \epsilon^\lambda_{ir} \epsilon^\lambda'_{sr} W(\lambda'1s;\lambda \lambda' \sigma),
\end{equation}

Here, the sum over $t$ goes to particle ($t = p$) or hole ($t = h$) states when $(rs) = (hh')$ or $(rs) = (pp')$, respectively.

The formal analogy between the structure of the phonon matrix $A^\rho(\lambda\alpha,\lambda'\alpha')$ and the form of the TDA matrix $A^q(\phi\rho,\phi'\rho')$ was pointed out [56]. The first is deduced from the second by replacing the p-h energies with the sum of phonon energies and the p-h interaction with a phonon-phonon interaction.

Equation (12) is not an eigenvalue equation yet. We have first to expand the amplitudes $X$ [Eq. (13)] in terms of the expansion coefficients $C^\rho_{\lambda\alpha}$ of the states $|n;\beta\rangle$ [Eq. (8)] obtaining
\begin{equation}
X^\rho_{\lambda\alpha} = \sum_{\lambda'\alpha'} D^\rho(\lambda\alpha,\lambda'\alpha')C^\rho_{\lambda'\alpha'},
\end{equation}

where
\begin{equation}
D^\rho(\alpha\lambda,\alpha'\lambda') = [(n - 1,\alpha) \times O_{\lambda}\mid O^1_{\lambda'} \times (n - 1,\alpha')].
\end{equation}
is the metric matrix. Upon insertion of the expansion (19) into Eq. (12), we get the generalized eigenvalue equation
\begin{equation}
\sum_{\lambda'\alpha'} \mathcal{H}^\beta(\lambda\alpha,\lambda'\alpha')C^\beta_{\lambda'\alpha'} = \sum_{\lambda'\alpha'} (A^\rho D^\beta(\lambda\alpha,\lambda'\alpha')C^\beta_{\lambda'\alpha'} = E_\beta \sum_{\lambda'\alpha'} D^\beta(\lambda\alpha,\lambda'\alpha')C^\beta_{\lambda'\alpha'}.
\end{equation}

This is effectively the representation of the eigenvalue equation
\begin{equation}
\mathcal{H}C = (A^\rho D^\beta)C = EC
\end{equation}
in the overcomplete basis $O^{1}_{\lambda} \times (n - 1,\alpha)|\beta\rangle$.

The redundant states are eliminated by the procedure outlined in Refs. [54,55], based on the Cholesky decomposition method. This method selects a basis of linear independent states $O^{1}_{\lambda} |n - 1,\alpha\rangle$ spanning the physical subspace of the correct dimensions $N_n < N_r$ and, thus, enables us to construct a $N_n \times N_n$ nonsingular matrix $\mathcal{D}_n$. By left multiplication in the $N_n$-dimensional subspace we get from Eq. (22)
\begin{equation}
[\mathcal{D}_n^{-1}(A^\rho D^\beta)]C = EC.
\end{equation}

This equation determines only the coefficients $C^\rho_{\lambda\alpha}$ of the $N_n$-dimensional physical subspace. The remaining redundant $N_r - N_n$ coefficients are undetermined and, therefore, can be safely put equal to zero. The eigenvalue problem within the $n$-phonon subspace is thereby solved exactly.

Since recursive formulas hold for all quantities entering $A$ and $D$, the eigenvalue equations are solved iteratively starting from the TDA phonons and, thereby, yield a set of orthonormal multiphonon states $|0\rangle, |1\lambda\rangle, \ldots |n;\alpha\rangle, \ldots$.

In such a basis, the Hamiltonian matrix is composed of a sequence of diagonal blocks, one for each $n$, mutually coupled by off-diagonal terms $\langle n'| H \mid n\rangle$ which are nonvanishing only for $n' = n \pm 1, n \pm 2$ and are computed by means of recursive formulas. A matrix of such a simple structure can be easily diagonalized yielding eigenfunctions of the form
\begin{equation}
|\Psi_n\rangle = \sum_{n\alpha} C^{(n)}_{\alpha}|n;\alpha\rangle.
\end{equation}

The eigenstates so obtained may be used to compute the transition amplitudes. In the coupled scheme, the one-body operator has the form
\begin{equation}
\mathcal{M}(\lambda) = \frac{1}{[\lambda]^{1/2}} \sum_{rs} (r||\mathcal{M}(\lambda)||s)[a^\dagger_r \times b_s]^\lambda.
\end{equation}
The reduced transition amplitudes are given by
\begin{equation}
\langle \Psi_{fJ_f} \parallel \mathcal{M}(\lambda) \parallel \Psi_{iJ_i}\rangle = \sum_{(n;\alpha)\alpha'\beta} C_{\alpha}^{(i)} C_{\beta}^{(f)} \langle n_f,\beta J_f \parallel \mathcal{M}(\lambda) \parallel n_i,\alpha J_i\rangle,
\end{equation}

where the matrix elements of $\mathcal{M}_\lambda$ between multiphonon states are
\begin{equation}
\langle n_f,\beta J_f \parallel \mathcal{M}(\lambda) \parallel n_i,\alpha J_i\rangle
= \delta_{\lambda,i} \delta_{\lambda;i}(\lambda)\delta_{\lambda,i}(\lambda) \mathcal{M}[0 \rightarrow (\lambda\lambda)]X^\lambda_{\lambda',\lambda'}
+ \delta_{\lambda,i} \delta_{\lambda;i}(\lambda)\delta_{\lambda,i}(\lambda) \mathcal{M}[0 \rightarrow (\lambda\lambda)]X^\lambda_{\lambda',\lambda'}
+ \delta_{\lambda,i} \delta_{\lambda;i}(\lambda)\delta_{\lambda,i}(\lambda) \mathcal{M}[0 \rightarrow (\lambda\lambda)]X^\lambda_{\lambda',\lambda'}
+ \delta_{\lambda,i} \delta_{\lambda;i}(\lambda)\delta_{\lambda,i}(\lambda) \mathcal{M}[0 \rightarrow (\lambda\lambda)]X^\lambda_{\lambda',\lambda'}.
\end{equation}

The last is a scattering term where states with the same number of phonons are coupled through the single-particle transition matrix elements $\langle r||\mathcal{M}_\lambda||s\rangle$, weighted by the particle or hole
where the NNLOopt overestimates the binding energy per nucleon parametrization. However, as mentioned in the introduction, other interactions, including chiral potentials with different small.

contribution to some low-lying states in O and Ca isotopes is 3NF. Moreover, a preliminary analysis has shown that the 3NF medium-light nuclei were reproduced without introducing any several bulk as well as spectroscopic properties of light and T

amplitudes different numbers of phonons and contain the TDA transition density matrices. The other two terms connect states with

\[ T_{2} = \frac{1}{2m A} \sum_{i \neq j} \vec{p}_i \cdot \vec{p}_j \]  

is a two-body piece which is incorporated into the potential V. The full Hamiltonian is therefore  

\[ H = T + V, \]  

dealing with low-energy dipole spectra, it is of great importance to obtain 1− states free of spurious admixtures induced by the c.m. excitation. We achieve this task for the TDA 1− phonons by the method outlined in Ref. [52] based on the Gram-Schmidt orthogonalization method. Let us define the c.m. spurious state as  

\[ |\varphi_{0}\rangle = \frac{1}{N_1} R_{\mu}|0\rangle, \]  

where \( R_{\mu} \) is the c.m. coordinate and \( N_1 \) the normalization constant. Expanded in the p-h basis states \(|i\rangle = (|p \times h^{-1}\rangle)^{T}\), it acquires the structure  

\[ |\varphi_{0}\rangle = \frac{1}{N_1} R_{\mu}|0\rangle = \frac{1}{N_1} \sum_{i=1}^{n} C_i|i\rangle, \]  

where  

\[ C_i = C_{\phi h}^{0} = \sqrt{\frac{4\pi}{9}} \frac{1}{A} \langle p|r Y_{i}\rangle |h \rangle \]  

are unnormalized expansion coefficients. The normalization factor is obtained from  

\[ N_{1}^{2} = \sum_{i=1}^{n} |C_i|^{2}. \]  

The Gram-Schmidt procedure yields orthogonalized states of the form  

\[ |\varphi_{k-1}\rangle = \frac{1}{N_{(k-1)}N_{k}} \left[ N_{(k-1)}^{2}|k-1\rangle - \sum_{i=k,n} C_{k-1} C_i|i\rangle \right], \]  

where \( k = 2,3,\ldots,n \) and  

\[ N_{k}^{2} = \sum_{i=k,n} |C_i|^{2}. \]  

For \( k = n \) the sum \( \sum_{i=k,n} \) disappears. So we have simply  

\[ |\varphi_{n-1}\rangle = \frac{1}{N_{(n-1)}N_{n}} \left[ N_{n}^{2}|n-1\rangle - C_{n-1} C_n |n\rangle \right], \]  

where  

\[ N_{n}^{2} = |C_n|^{2}. \]
The basis states so obtained are adopted to construct the Hamiltonian matrix \( \{ \langle \phi_r | H | \phi_s \rangle \} \). Its diagonalization yields eigenstates rigorously free of spurious admixtures induced by the c.m. excitation. These eigenstates, which are given in terms of the orthogonalized states \( | \phi_r \rangle \), recover the standard TDA structure by expressing the states \( | \phi_r \rangle \) in terms of the \( (|p \times \hbar|^2) \) states.

The EMPM states built of these c.m. spurious free TDA phonons are also spurious free. They are linear combinations of products of phonons. The exchange terms between fermions entering two different phonons of a product are taken into account through the metric matrix \( D \) which leaves the phonon structure unchanged.

**C. Dipole response**

We consider the \( E1 \) strength function

\[
S(E_\lambda, \omega) = \sum_v B_v(E_\lambda) \delta(\omega - \omega_v)
\]

\[
\approx \sum_v B_v(E_\lambda) \rho_\Delta(\omega - \omega_v).
\]  

(45)

Here \( \omega \) is the energy variable, \( \omega_v \) the energy of the transition of multipolarity \( E_\lambda \) from the ground to the \( v \)th excited state of spin \( J = \lambda \), and

\[
\rho_\Delta(\omega - \omega_v) = \frac{\Delta}{2\pi} \frac{1}{(\omega - \omega_v)^2 + \left( \frac{\Delta}{2} \right)^2}
\]  

(46)

is a Lorentzian of width \( \Delta \), which replaces the \( \delta \) function as a weight of the reduced strength

\[
B_v(E_\lambda) = | \langle v, \lambda | M(E_\lambda) | 0 \rangle |^2.
\]  

(47)

The \( E_\lambda = E1 \) operator is

\[
M(E1 \mu) = \frac{e}{2} \sum_{i=1}^{A} (1 - r_3^i) r_1^i Y_{1\mu}(\hat{r}_i).
\]  

(48)

where \( r_3^i = 1 \) for neutrons and \( r_3^i = -1 \) for protons.

The total \( E1 \) cross section is determined by the strength function and is given by

\[
\sigma = \int_0^\infty \sigma(\omega) d\omega = \frac{16\pi^3}{9hc} \int_0^\infty \omega S(E1, \omega) d\omega
\]  

(49)

or, after integration, by

\[
\sigma = \frac{16\pi^3}{9hc} m_1,
\]  

(50)

where

\[
m_1 = \sum_n \omega_n B_n(E1)
\]  

(51)

is the first momentum. If one neglects momentum dependent and exchange terms in the Hamiltonian, \( m_1 \) fulfills the classical energy weighted Thomas-Reiche-Kuhn (TRK) sum rule

\[
m_1 = \frac{\hbar^2}{2m} \frac{9}{4\pi} \frac{NZ}{A} e^2
\]  

(52)

and the total cross section assumes the value

\[
\sigma = (2\pi)^2 \frac{\hbar^2}{2m \hbar c} \frac{e^2}{A} = 60 \frac{NZ}{A} \text{ (MeV mb)}.
\]  

(53)

An observable sensitive to the low-energy spectrum is the dipole polarizability \([29,47]\)

\[
\alpha_D = \frac{8\pi}{9} m_{-1},
\]  

(54)

where

\[
m_{-1} = \sum_n \omega_n^{-1} B_n(E1)
\]  

(55)

is the \( E1 \) inverse moment. This quantity links the dipole polarizability to the cross section \( \sigma(\omega) \) according to the relation \([20]\)

\[
\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int_0^\infty \omega^{-2} \sigma(\omega) d\omega.
\]  

(56)

For a more complete investigation of the nature of the low-lying \( E1 \) levels, we compute also the isoscalar dipole transition strength using the operator

\[
M_{IS}(\lambda = 1, \mu) = \sum_{i=1}^{A} r_3^i Y_{1\mu}(\hat{r}_i).
\]  

(57)

It is to be noticed the absence of the corrective term generally added to remove, partially, the c.m. contribution to the transition strength. Such a term is ineffective in our case.

**IV. NUMERICAL CALCULATIONS AND RESULTS**

**A. Preliminary HF and TDA analysis**

The optimized chiral potential NNLOopt, with the addition of the density dependent term \( V_\rho \), is adopted to generate a HF basis in a configuration space which includes 13 harmonic oscillator major shells, up to the principal quantum number \( N_{\text{max}} = 12 \). This space is sufficient for reaching a good convergence of the single-particle spectra below and around the Fermi surface. In going, for instance, from \( N_{\text{max}} = 9 \) to \( N_{\text{max}} = 12 \), step by step, the energies change at most by \( \sim 0.1 \text{ MeV} \) at each step. More appreciable variations with \( N_{\text{max}} \) are noticed in the spectrum far above the Fermi surface, a general feature of HF. They, however, induce some fluctuations only in the high energy sector of the TDA strength distribution, which are wiped out by the coupling with the two-phonon states. The low-energy dipole spectrum reaches convergence very fast even at the TDA level. The rate of convergence for both HF and TDA is the same whether or not we add \( V_\rho \) to NNLOopt.

Among the HF states so obtained, those belonging to three major shells above and three below the Fermi surface are used to construct the p-h space and, then, generate the TDA phonons through the diagonalization of the nuclear Hamiltonian.

NNLOopt improves the description of the TDA dipole response as compared to the \( V_{\text{limk}} \) potential, deduced from CD-Bonn \([52]\), or to other realistic effective interactions \([51]\). As shown in Fig. 1, in fact, it yields a strength distribution peaked around \( \sim 23 \text{ MeV} \), much lower than the peak energy (\( \sim 45 \text{ MeV} \)) obtained by using \( V_{\text{limk}} \) (Fig. 3(c) of Ref. \([52]\)).
Its better performance is to be ascribed to the more pronounced compression of the proton and neutron single-particle spectra. These are shown in Fig. 2 and should be compared with the neutron spectrum produced for the same nucleus by the CD-Bonn potential (Fig. 1(b) of Ref. [52]).

The main peaks are still a few MeV above the measured GDR peak at ~16 MeV [6], hence the need to add $V_\rho$, though with a smaller coupling constant. Indeed, a value of $C_\rho \sim 1000$ MeV fm$^6$ is sufficient to shift them down in the experimental GDR region (Fig. 1). In Ref. [52] we had to use $C_\rho \sim 4200$ MeV fm$^6$. It might be worth pointing out that this is the only free parameter we used in the calculation.

The shift promoted by $V_\rho$ is clearly due to the further compression of the single-particle spectrum, as shown in Fig. 2. The spectrum so obtained still deviates in several important details from the empirical one [65]. The levels within a major shell are not sufficiently packed as compared to the empirical data while the spin-orbit intruders do not approach sufficiently the levels of the major shells below. These discrepancies affect the fine structure of the resonance rather than its position.

### B. EMPM results

We determine the EMPM $1^-$ states by diagonalizing the Hamiltonian in a space spanned by one- $(|\lambda,1^-\rangle)$ plus two-phonon $(|(n=2)\beta,1^-\rangle)$ basis states. The two-phonon states are generated in a restricted subspace. We include, in fact, the states $|\lambda_1 \times \lambda_2,1^\pm\rangle \equiv \{|O_{\lambda_1}\times |\lambda_2,1^\pm\rangle\}$ of energies $E_{\lambda_1} + E_{\lambda_2} \lesssim 30$ MeV to construct and diagonalize the matrix $AD$, yielding thereby the basis states $(|n=2)\beta,1^-\rangle$.

An analogous truncating criterion is adopted to generate the two-phonon basis states $(|(n=2)\beta,0^+\rangle)$ to be added to the HF vacuum $|0\rangle$ and the $0^+$ TDA states in order to determine the ground state.

This gets depressed by $\Delta E = 10.5$ MeV with respect to the unperturbed HF state because of the strong coupling between the vacuum and the two-phonon space. It becomes also strongly correlated. Its two-phonon components account for ~23% of the wave function, a result analogous to the ones obtained for $^{16}$O [56] and $^{208}$Pb [57] and consistent with shell model calculations on $^{208}$Pb [14,66].

This ground state energy shift would spoil the description of the dipole response by pushing the strength at too high energy, unless we add the three-phonon basis states to determine the $1^-$ eigenstates. It is in fact known from EMPM calculations on $^{16}$O [56] that the three-phonon configurations couple strongly to the $1^-$ TDA phonons and, thereby, counterbalance the coupling between ground and two-phonon states by pushing the strength back to the experimental region.

Since including the three-phonon basis in a heavy nucleus like $^{132}$Sn is practically impossible, we neglect for consistency the ground state correlations and adopt the HF vacuum as the ground state. This assumption was already made for $^{208}$Pb [57] and is consistent with the one made in shell model calculations [14,66].

Thus, the two-phonon states are included only to determine the $1^-$ states. These have the form (24) and are used to evaluate the dipole transition strengths and cross sections.

Figure 3 shows the TDA and EMPM cross sections computed using a Lorentzian of width $\Delta = 0.5$ MeV. The EMPM cross section is severely quenched and reshaped due to the one- to two-phonon coupling. It has a smoother behavior with respect to TDA and follows more closely the experimental points. It must be kept in mind, however, that the error bars are very large, especially in the high energy sector. Moreover, we do not fold our theoretical curve with the experimental acceptance filter [6] necessary for a more correct comparison with the data, as done, for instance, in Ref. [42].

The first moment $m_1$ remains practically unchanged in going from TDA to the EMPM. In both approaches, it overestimates the TRK sum rule by a factor of ~1.7. Also the inverse moment $m_{-1}$ changes little. It yields for the dipole polarizability the value $\alpha_D = 10.82$ fm$^3$, slightly larger than the empirical estimate $\alpha_D = 10.081 \pm 0.150$ fm$^3$ extrapolated for $^{132}$Sn [49] from the $^{208}$Pb data [20].
Both TDA and EMPM calculations yield a non-negligible strength below $\sim 12$ MeV which gives rise to a small peak in the cross section around $\sim 10$ MeV, fairly close in position and height to the one at $\sim 9.8$ MeV observed experimentally [6]. The fraction of the TRK sum rule exhausted by the low-lying states up to $\sim 11$ MeV is $\sim 6.5\%$ in TDA and $\sim 5.8\%$ in the EMPM. Both values are within the (large) error of the corresponding measured summed strength $4(3)\%$ [6].

An additional effect of the phonon coupling is the fragmentation of the strength. This effect, partly hidden in the cross section due to the smoothing action of the Lorentzian, is clearly visible in the $E1$ spectra shown in Fig. 4. The strength splits into a low and a GDR sector in both TDA and EMPM. The GDR EMPM spectrum is much more dense and is composed of peaks of considerably shorter height as compared to TDA. However, the TDA and EMPM spectra look very similar in the low-energy sector. It seems, therefore, that the phonon coupling produces only quenching but not fragmentation in this region.

A closer look, however, shows that this is not the case. If we amplify the scale, as done in Fig. 5, the EMPM spectrum is much more rich and is composed of a large number of weakly excited levels, not present in TDA. Though most of the levels lie just above the neutron threshold, the spectrum resembles the discrete ones produced by ($\gamma,\gamma'$) and ($\alpha,\alpha'\gamma$) experiments in the open shell $^{124}$Sn [7,18,19], whose comparative analysis has led to the suggestion of a splitting of the dipole strength into a low-lying isoscalar and a high-lying isovector branch [17].

As shown in Fig. 6, indeed, also the isoscalar probe ($^{57}$) excites the low-lying EMPM states. However, the computed isoscalar and isovector spectra overlap over the entire low-energy region. Thus, the present calculation does not predict a splitting between isoscalar and isovector dipole modes at low energy for $^{132}$Sn.
It remains to investigate the nature of the states. As shown in Fig. 7, the transition density pertaining to the most strongly excited low-lying $1^-$ state shows a neutron excess for large values of the radial coordinate, suggesting the pygmy nature of such a state. This behavior is to be contrasted with the one exhibited by the transition to the most strongly excited $1^-$ state in the region of the GDR, which clearly describes an oscillation of protons versus neutrons.

A further insight into the nature of the low-lying states is gained by investigating their phonon composition and the structure of the dominant phonons. Among the lowest 20 states shown in Table I, fewer than half are dominated by the one-phonon components while the others are mainly two-phonon states. The latter states are weakly excited.

![FIG. 7](Color online) $E1$ transition density for (a) a low-lying $1^-$ state and (b) one in the GDR region.

Table II shows the p-h composition of two typical phonons contributing to the strength in the low-energy and GDR regions, respectively. Both low-lying and GDR phonons are built of a large number of p-h excitations and, therefore, are seemingly collective. However, they have distinctive p-h structures. The low-lying phonon is composed predominantly of neutron excitations around the neutron decay threshold. These strong excitations are coexisting with few weakly excited levels close to the experimental GDR bump.

In our case, we simply added a phenomenological, density dependent term and determined its coupling constant so as to get some overlap with the experimental GDR bump. This $ad hoc$ prescription is certainly unsatisfactory. It might, nonetheless, offer some useful hint on how to proceed in order to derive the necessary corrective pieces to NNLO$_{opt}$ within a general and consistent approach.

The other result emerging from the EMPM is that the inclusion of the two-phonon basis states induces a strong fragmentation of both GDR and PDR. At low energy, a large number of weakly excited levels coexist with few strong excitations around the neutron decay threshold. These

### Table I. Phonon composition of the lowest 20 $1^-$ states.

| $J^+$ | $\omega_0$ (MeV) | $|C_1^{(0)}|^2$ | $|C_2^{(0)}|^2$ |
|-------|-----------------|-----------------|-----------------|
| $1^-$ | 5.09581         | 0.11502         | 0.88498         |
| $1_2^-$| 7.30528         | 0.92385         | 0.07615         |
| $1_3^-$| 7.85679         | 0.89871         | 0.10129         |
| $1_4^-$| 7.83090         | 0.90887         | 0.09113         |
| $1_5^-$| 7.93351         | 0.89186         | 0.10494         |
| $1_6^-$| 7.98106         | 0.14354         | 0.95646         |
| $1_7^-$| 8.50946         | 0.11299         | 0.98871         |
| $1_8^-$| 8.52024         | 0.03015         | 0.96985         |
| $1_9^-$| 8.96778         | 0.79747         | 0.20253         |
| $1_{10}^-$| 9.17765        | 0.00064         | 0.99936         |
| $1_{11}^-$| 9.23154        | 0.05044         | 0.94956         |
| $1_{12}^-$| 9.30731        | 0.03582         | 0.96418         |
| $1_{13}^-$| 9.32758        | 0.00960         | 0.99040         |
| $1_{14}^-$| 9.46252        | 0.02983         | 0.97017         |
| $1_{15}^-$| 9.54972        | 0.00701         | 0.99299         |
| $1_{16}^-$| 9.61935        | 0.76969         | 0.23031         |
| $1_{17}^-$| 9.74903        | 0.82698         | 0.17302         |
| $1_{18}^-$| 9.80037        | 0.03210         | 0.96790         |
| $1_{19}^-$| 9.86425        | 0.00475         | 0.99525         |
| $1_{20}^-$| 9.92197        | 0.01055         | 0.98945         |

### Table II. Particle-hole composition of two selected phonons.

<table>
<thead>
<tr>
<th>(p(h)$^{-(1)} v$</th>
<th>$C_{p(h)}^0$</th>
<th>(p(h)$^{-(1)} v$</th>
<th>$C_{p(h)}^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0g_{7/2}(0f_{5/2})^{-1}$</td>
<td>$-0.08229$</td>
<td>$2p_{1/2}(1d_{3/2})^{-1}$</td>
<td>$0.11501$</td>
</tr>
<tr>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$0.11867$</td>
<td>$2p_{1/2}(2s_{1/2})^{-1}$</td>
<td>$0.85924$</td>
</tr>
<tr>
<td>$1_f_{7/2}(1d_{3/2})^{-1}$</td>
<td>$0.08687$</td>
<td>$2p_{1/2}(1d_{3/2})^{-1}$</td>
<td>$-0.16220$</td>
</tr>
<tr>
<td>$1_f_{7/2}(2s_{1/2})^{-1}$</td>
<td>$0.37892$</td>
<td>$2p_{1/2}(2s_{1/2})^{-1}$</td>
<td>$-0.05794$</td>
</tr>
<tr>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$-0.09230$</td>
<td>$0f_{7/2}(1d_{3/2})^{-1}$</td>
<td>$0.17728$</td>
</tr>
<tr>
<td>$0f_{7/2}(0f_{5/2})^{-1}$</td>
<td>$0.25438$</td>
<td>$1f_{7/2}(0g_{9/2})^{-1}$</td>
<td>$0.14461$</td>
</tr>
<tr>
<td>$1d_{5/2}(0f_{5/2})^{-1}$</td>
<td>$-0.14285$</td>
<td>$1f_{7/2}(0g_{9/2})^{-1}$</td>
<td>$-0.05472$</td>
</tr>
<tr>
<td>$1d_{5/2}(0f_{5/2})^{-1}$</td>
<td>$-0.06973$</td>
<td>$1f_{7/2}(0g_{9/2})^{-1}$</td>
<td>$-0.11795$</td>
</tr>
<tr>
<td>$1d_{5/2}(1p_{3/2})^{-1}$</td>
<td>$0.07800$</td>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$0.10989$</td>
</tr>
<tr>
<td>$1d_{5/2}(1p_{3/2})^{-1}$</td>
<td>$0.08258$</td>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$0.51199$</td>
</tr>
<tr>
<td>$2s_{1/2}(1p_{3/2})^{-1}$</td>
<td>$-0.34226$</td>
<td>$3p_{3/2}(1d_{3/2})^{-1}$</td>
<td>$-0.11688$</td>
</tr>
<tr>
<td>$2s_{1/2}(1p_{3/2})^{-1}$</td>
<td>$-0.06826$</td>
<td>$3p_{3/2}(1d_{3/2})^{-1}$</td>
<td>$0.08979$</td>
</tr>
<tr>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$0.33378$</td>
<td>$1f_{7/2}(0g_{9/2})^{-1}$</td>
<td>$-0.40422$</td>
</tr>
<tr>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$-0.21556$</td>
<td>$0h_{1/2}(0g_{9/2})^{-1}$</td>
<td>$-0.21556$</td>
</tr>
</tbody>
</table>
levels are excited by both isoscalar and isovector probes and, therefore, seem to be the analog of the spectra detected in the open shell $^{124}$Sn. The calculation, however, does not produce a clear splitting between isoscalar and isovector strengths.

The analysis of the transition densities suggests that the strong transitions to the states of the GDR region are induced by oscillations of protons versus neutrons, while the ones to the low-lying states are mainly promoted by the neutron skin, thereby suggesting the pygmy nature of this low-energy mode. The two pictures find further support in the p-h composition of the dominant phonons entering the states in the two regions. It is, nonetheless, to be pointed out that more than half of the states describing the low-lying weakly excited levels have dominant two-phonon components.

It is of great interest to investigate if and to what extent the NNLO$_{pt}$ potential improves the description of the dipole and other multipole responses in other nuclei. An important test may be provided by the stable $^{208}$Pb, which offers fairly complete and accurately measured data.

Such a study is to be carried out within a theoretical framework which adopts a self-consistent basis and takes into due account the anharmonicities induced by the coupling of the TDA or RPA phonons to more complex configurations.

As pointed out already, we neglect the two-phonon ground state correlations and adopt the HF vacuum as the ground state. The action of the two-phonon coupling to the HF vacuum should be offset by the coupling of the one-phonon to the three-phonon $1^-$ basis states. Unfortunately, enlarging the multiphonon space so as to include the three-phonon subspace is practically impossible in heavy nuclei without a severe cut of the phonon basis. The condition for a reliable truncation is that the coupling involves a very restricted subset of three-phonon states. At present, this issue can be settled only for light nuclei like $^{16}$O. We plan to use this nucleus as a laboratory in order to explore if there is any chance of truncating severely the multiphonon space with marginal detriment to the accuracy of the calculation.

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