Electric and Vorticity Strengths in Heavier Nuclei

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1. **Motivation and brief formulation of the Separable RPA approach**

2. **Different Skyrme parametrizations are analyzed from the point of view of the photoabsorption cross section**

3. **Photoabsorption cross section in the Pigmy region is discussed**

4. **Vorticity multipole operator strength function is introduced as a measure of the irrotationality of a nuclear matter**

5. **Some preliminary results of the vorticity strength is presented**

Effective n-n interactions (Skyrme , Gogny, relativistic mean field) are widely used for the description of the static characteristics of spherical and deformed nuclei

Dynamics of small amplitude vibrations is mainly described by the RPA. However, for heavy nuclei the standard RPA method requires the construction and diagonalization of huge matrices. RPA problem becomes simpler if the residual two-body interaction in the nuclear Hamiltonian is factorized as a product of two s.p. operators (see e.g. P.Ring, P.Schuck, The Nuclear Many-Body Problem, Springer N.Y. (1980)).

$$\hat{H} = \hat{h}_{HFB} + \hat{V}_{res}$$

$$\hat{V}_{res} = -\frac{1}{2}\sum_{k}\sum_{k'}\left\{ \kappa_{kk'}\,\hat{X}_k^{(1)}\,\hat{X}_{k'}^{(1)} + \eta_{kk'}\,\hat{Y}_k^{(1)}\,\hat{Y}_{k'}^{(1)} \right\} \qquad k = 1,\cdots,K$$

where $\hat{X}_k^{(1)}$ and $\hat{Y}_k^{(1)}$ are two-quasiparticle parts of s.p. operators

$$\hat{X}_k = \sum_{ij} < i\,|\,\hat{X}_k\,|\,j > a_i^+ a_j \qquad T\,\hat{X}_k\,T^{-1} = \hat{X}_k$$

$$\hat{Y}_k = \sum_{ij} < i\,|\,\hat{Y}_k\,|\,j > a_i^+ a_j \qquad T\,\hat{Y}_k\,T^{-1} = -\hat{Y}_k$$

We developed a general self-consistent separable RPA (SRPA) approach applicable to any density- and current- dependent functional - see e.g.

sperical nuclei: V.O.Nesterenko, J.Kvasil, P.-G.Reinhard, Phys.Rev. C66, 044307 (2002)

deformed nuclei: V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Vesely, P.-G.Reinhard Phys.Rev. C74, 064306 (2006)

for the determination of operators $\hat{X}_k^{(1)}$ and $\hat{Y}_k^{(1)}$ by fully self-consistent method starting from the general energy functional

$$E = \langle HFB | \hat{H} | HFB \rangle = \int \mathcal{H}(J_\alpha(\vec{r}))\, d^3 r$$

with

$$J_\alpha(\vec{r}) = \langle HFB | \hat{J}_\alpha(\vec{r}) | HFB \rangle$$

some $\hat{J}_\alpha(\vec{r})$ are time-even and some are time-odd

Basic idea of the SRPA method:

nucleus is excited by external s.p. fields $(\hat{Q}_k, \hat{P}_k) \quad k = 1, \cdots, K$:

$$\hat{Q}^+_{\tau k} = \hat{Q}_{\tau k} \quad ; \quad T \hat{Q}_{\tau k} T^{-1} = \hat{Q}_{\tau k} \quad ; \quad [\hat{H}, \hat{Q}_{\tau k}] = -i \hat{P}_{\tau k}$$

$$\hat{P}^+_{\tau k} = \hat{P}_{\tau k} \quad ; \quad T \hat{P}_{\tau k} T^{-1} = -\hat{P}_{\tau k} \quad ; \quad [\hat{H}, \hat{P}_{\tau k}] = -i \hat{Q}_{\tau k}$$

$$\hat{Q}_k = \sum_i r_i^l Y_{\lambda\mu}(i)$$ for electric type excitation

$$\hat{P}_k = \sum_i r_i^{l'} [\vec{\sigma} \otimes Y_l]_{\lambda\mu}$$ for magnetic type excitation

Using TDHFB with the linear response theory we obtain : $\hat{H} = \hat{h}_{HFB} + \hat{V}_{res}$

$$\hat{h}_{HFB} = \int d^3r \sum_{\alpha_+} [\frac{\partial E}{\partial J_{\alpha_+}(\vec{r})}] \hat{J}_{\alpha_+}(\vec{r})$$

$$\hat{V}_{res}^{(sep)} = \frac{1}{2} \sum_{k,k'=1}^{K} \left\{ \kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_{k'} \right\}$$

$$\hat{X}_k = i \int d^3r \int d^3r' \sum_{\alpha_+ \alpha'_+} [\frac{\partial^2 E}{\partial J_{\alpha_+}(\vec{r}') \, \partial J_{\alpha'_+}(\vec{r})}] < | [\hat{P}_k, \hat{J}_{\alpha_+}(\vec{r})] | > \hat{J}_{\alpha'_+}(\vec{r}')$$

$$\hat{Y}_k = i \int d^3r \int d^3r' \sum_{\alpha_- \alpha'_-} [\frac{\partial^2 E}{\partial J_{\alpha_-}(\vec{r}') \, \partial J_{\alpha'_-}(\vec{r})}] < | [\hat{Q}_k, \hat{J}_{\alpha_-}(\vec{r})] | > \hat{J}_{\alpha'_-}(\vec{r}')$$

$$T \hat{J}_{\alpha_+} T^{-1} = \hat{J}_{\alpha_+}, \; T \hat{J}_{\alpha_-} T^{-1} = -\hat{J}_{\alpha_-} \;\; \Rightarrow \;\; T \hat{X}_k T^{-1} = \hat{X}_k, \; T \hat{Y}_k T^{-1} = -\hat{Y}_k$$

where strength constant matrixes are

$$\kappa_{kk'}^{-1} = \int d^3r \int d^3r' \sum_{\alpha_+ \alpha'_+} < | [\hat{P}_k, \hat{J}_{\alpha_+}(\vec{r})] | > [\frac{\partial^2 E}{\partial J_{\alpha'_+}(\vec{r}') \, \partial J_{\alpha_+}(\vec{r})}] < | [\hat{P}_{k'}, \hat{J}_{\alpha'_+}(\vec{r}')] | >$$

$$\eta_{kk'}^{-1} = \int d^3r \int d^3r' \sum_{\alpha_- \alpha'_-} < | [\hat{Q}_k, \hat{J}_{\alpha_-}(\vec{r})] | > [\frac{\partial^2 E}{\partial J_{\alpha'_-}(\vec{r}') \, \partial J_{\alpha_-}(\vec{r})}] < | [\hat{Q}_{k'}, \hat{J}_{\alpha'_-}(\vec{r}')] | >$$

RPA equations:

$$[H, O_\nu^+] = \omega_\nu O_\nu^+ \qquad [H, O_\nu] = -\omega_\nu O_\nu \qquad [O_\nu, O_{\nu'}^+] = \delta_{\nu\nu'}$$

gives energies, forward and backward amplitudes of phonon operator

$$O_\nu^+ = \sum_{ij} \left\{ \psi_{ij}^{(\nu)} b_{ij}^+ - \varphi_{ij}^{(\nu)} b_{ij} \right\} \longleftrightarrow \omega_\nu$$

RPA equations with the separable residual interactions can be transferred into the homogeneous system of algebraic equations. Dimension of the matrix of this system is given by the number of s.p. operators $\hat{X}_k$ and $\hat{Y}_k$ in the residual interaction. Detailed description of our SRPA method can be found in the papers:

W.Kleinig, V.O.Nesterenko, J.Kvasil, P.-G.Reinhard, P.Vesely, PRC78, 044315 (2008)
V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Vesely, P.-G.Reinhard, PRC74, 064306 (2006)

Knowing the structure of phonons we can calculate el.mg. reduced probability from the RPA ground state $|RPA>$ to one-phonon state $O_\nu^+|RPA>$ with the energy ω_ν

$$B(Z\lambda\mu, |RPA> \to |\nu>) = |< RPA|[O_\nu, M_{Z\lambda\mu}]|RPA>$$

$Z = el., mg.$ | $\lambda -$ transition multipolarity | $M_{Z\lambda\mu} -$ transition multipole operator

Then the energy weighted strength function is:

$$S_L(Z\lambda\mu\,;E)=\sum_{\nu} B(Z\lambda\mu\,;|\,RPA>\rightarrow|\,\nu>)\,\omega_\nu^L\;\delta(E-\omega_\nu)$$

$$\approx\sum_{\nu} B(Z\lambda\mu\,;|\,RPA>\rightarrow|\,\nu>)\,\omega_\nu^L\;\xi(E-\omega_\nu)$$

This quantity can be determined even without the solving the RPA equations for each individual phonon state $|\,\nu>=O_\nu^+\,|\,RPA>$ using the Cauchy theorem and the substitution

$$\delta(E-\omega_\nu)\quad\rightarrow\quad\xi(E-\omega_\nu)=\frac{1}{2\pi}\frac{1}{(E-\omega_\nu)^2+(\Delta/2)^2}$$

see

V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Vesely, P.-G.Reinhard, PRC74, 064306 (2006)

Knowing the reduced probability or strength function we can determine the photoabsorption cross section:

$$\sigma(E)=\frac{8\pi^3\alpha}{e^2}\sum_{\nu}\sum_{\lambda\mu}\frac{(E_\nu)^{2\lambda-1}}{(\hbar c)^{2\lambda-2}}\frac{1}{[(2\lambda+1)!!]^2}\frac{\lambda+1}{\lambda}$$

$$\left[B(E\lambda\mu;|0>\rightarrow|\nu>)+B(E\lambda\mu;|0>\rightarrow|\nu>)\right]\delta(E-E_\nu)$$

where $\alpha=\frac{1}{137}$ $\qquad$ $\mu_N=\frac{e\hbar}{2mc}\sim 0.105\ fm$

or in more detail:

$$\sigma(E)=\sigma_{E1}(E)+\sigma_{E2}(E)+\sigma_{E3}(E)+\sigma_{M1}(E)\ +\ldots$$

$$\sigma_{E1}(E)=0.402\,E\,S_{E1}(E)$$

$$\sigma_{E2}(E)=3.089\,10^{-7}\,E^3\,S_{E2}(E)$$

$$\sigma_{E3}(E)=1.199\,10^{-13}\,E^5\,S_{E3}(E)$$

$$\sigma_{M1}(E)=4.437\,10^{-3}\,E\,S_{M1}(E)$$

$\sigma(E)$ in fm^2

$S_{E\lambda}(E)$ in $e^2\,fm^{2\lambda}\,MeV^{-1}$

$S_{M\lambda}(E)$ in $\mu_N^2\,fm^{2\lambda-2}\,MeV^{-1}$

E in MeV

We use the Skyrme energy density for the energy functional
- see e.g. J.Dobaczewski, J.Dudek, Phys.Rev. C52, 1827 (1995):

$$E(\rho,\tau,\vec{s},\vec{j},\vec{\mathfrak{J}},\vec{T},\chi)=\int \mathcal{H}(\vec{r})\, d^3r$$

with

$$\mathcal{H}(\vec{r})=\mathcal{H}_{kin}(\vec{r})+\mathcal{H}_{Sk}(\vec{r})+\mathcal{H}_{pair}(\vec{r})+\mathcal{H}_{Coul}(\vec{r})$$

$$\mathcal{H}_{kin}(\vec{r})=\frac{\hbar^2}{2m}\tau(\vec{r})$$

$$\mathcal{H}_{Coul}(\vec{r})=\frac{e^2}{2}\int d^3r'\,\rho_p(\vec{r})\frac{1}{|\vec{r}-\vec{r}'|}\rho_p(\vec{r}')-\frac{3}{4}e^2\left(\frac{3}{\pi}\right)^{1/3}[\rho_p(\vec{r})]^{4/3}$$

$$\mathcal{H}_{Sk}(\vec{r})=\sum_{t=0,1}\mathcal{H}_t^{(even)}+\sum_{t=0,1}\mathcal{H}_t^{(odd)}$$

$$\mathcal{H}_{pair}(\vec{r})=\frac{1}{4}\sum_{t=n,p}\chi_t^2\,V_t^{(0)}[1-(\frac{\rho}{\rho_{nm}})^{\gamma}]$$

$$\mathcal{H}_t^{(even)}(\vec{r})=C_t^{(\rho)}\rho_t^2+C_t^{(\Delta\rho)}\rho_t(\Delta\rho)+C_t^{(\tau)}\rho_t\tau_t+C_t^{(\mathfrak{J})}\vec{\mathfrak{J}}_t^2+C_t^{(\Delta\mathfrak{J})}\rho_t(\vec{\nabla}\vec{\mathfrak{J}}_t)$$

$$\mathcal{H}_t^{(odd)}(\vec{r})=C_t^{(s)}\vec{s}_t^2+C_t^{(\Delta s)}\vec{s}_t(\Delta\vec{s})+C_t^{(T)}\vec{s}_t\vec{T}_t+C_t^{(j)}\vec{j}_t^2+C_t^{(\Delta j)}\vec{s}_t(\vec{\nabla}\times\vec{j}_t)$$

$C_t^{(\rho)}(\rho), C_t^{(s)}(\rho), C_t^{(\Delta\rho)}, C_t^{(\tau)}, C_t^{(\mathfrak{J})},$
$C_t^{(\Delta\mathfrak{J})}, C_t^{(\Delta s)}, C_t^{(T)}, C_t^{(j)}, C_t^{(\Delta j)}, V_t^{(0)}, \gamma$ **interaction parameters**

$C_t^{(j)}=-C_t^{(\tau)}$

$C_t^{(T)}=-C_t^{(\mathfrak{J})}$ **gauge invariance**

$C_t^{(\Delta j)}=C_t^{(\Delta\mathfrak{J})}$

The dependence of the energy density $\mathcal{H}(\vec{r})$ on $\vec{r}$ goes through the following densities and currents:

density

$$\hat{\rho}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \psi_i^+(\vec{r})\, \psi_j(\vec{r})\, a_i^+ a_j$$

kinetic energy density

$$\hat{\tau}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \left(\vec{\nabla}\psi(\vec{r})\right) a_i^+ a_j$$

spin-orbit current

$$\hat{\vec{\mathfrak{I}}}(\vec{r}) = -\frac{i}{2} \sum_{\tau} \sum_{ij\in\tau} \left\{ \psi_i^+(\vec{r}) \left(\vec{\nabla}\times\psi_j(\vec{r})\right) + \left(\vec{\nabla}\times\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) \right\} a_i^+ a_j$$

current

$$\hat{\vec{j}}(\vec{r}) = \frac{i}{2} \sum_{\tau} \sum_{ij} \left\{ \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) - \psi_i^+(\vec{r}) \left(\vec{\nabla}\psi_j(\vec{r})\right) \right\} a_i^+ a_j$$

spin-current

$$\hat{\vec{s}}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \psi_i^+(\vec{r})\, \vec{\sigma}\, \psi_j(\vec{r})\, a_i^+ a_j$$

pairing density

$$\hat{\chi}_\tau(\vec{r}) = \sum_{i\in\tau} \psi_i^+(\vec{r})\, \psi_{\bar{i}}(r)\, (a_i^+ a_{\bar{i}}^+ + a_{\bar{i}} a_i)$$

kinetic energy – spin current

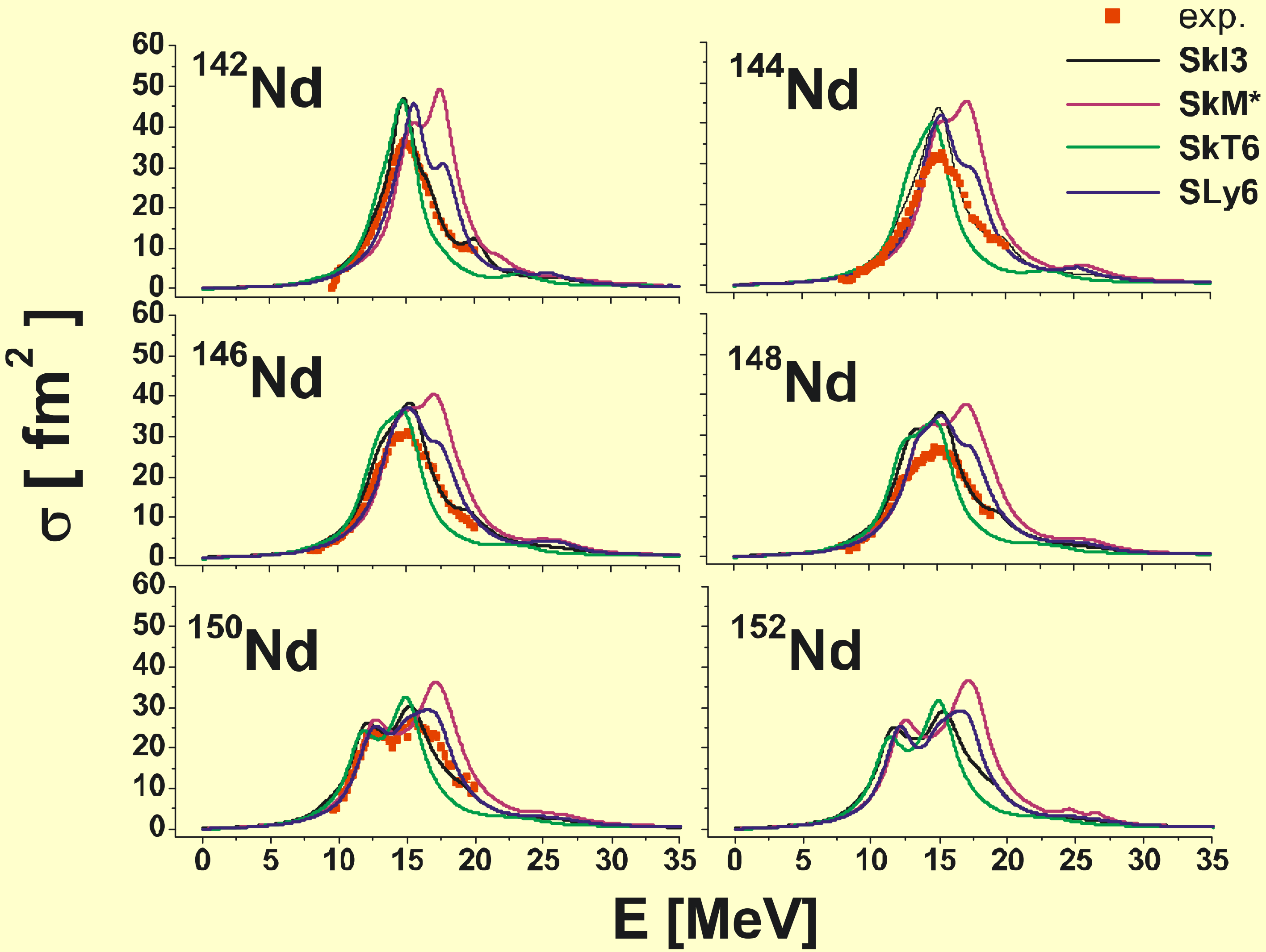
$$\hat{\vec{T}}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \vec{\sigma} \left(\vec{\nabla}\psi_j(\vec{r})\right) a_i^+ a_j$$

Comparison of experimental photoabsorption cross-section with calculated values for different Skyrme parametrizations

photoabsorption cross section gives possibility to test different parametrizations

exp. taken from P.Carlos et al. NPA 172, 437(1971)

similar results and similar agreement were obtained also for Mo, Sm, Sn izotopes



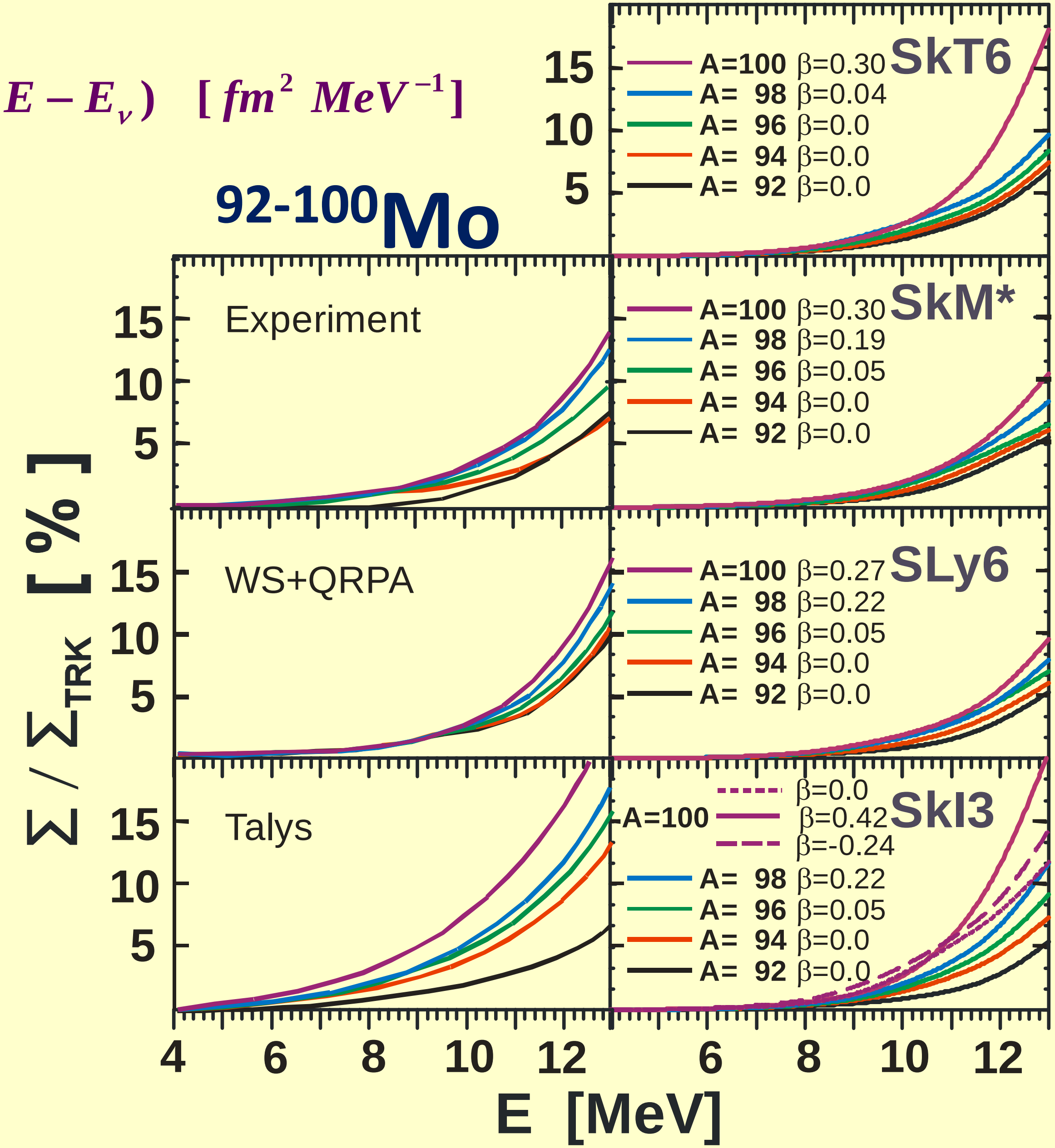
Cumulative integral photoabsorption cross section in the low-energy (Pigmy) region (4 - 13 MeV)

$$S_{E1}(E) = \sum_{\nu} B(E1; |\rangle \rightarrow |\nu\rangle)\, \rho_{\Delta}(E - E_{\nu}) \quad [\, fm^2\ MeV^{-1}]$$

$$\sigma_{E1}(E) \approx 4.01\, E\ S_{E1}(E) \quad [mb]$$

$$\Sigma = \int_4^E dE'\, \sigma_{E1}(E') \quad [mb\ MeV]$$

$$\Sigma_{TRK} \approx 60\, \frac{N\, Z}{A} \quad [mb\ MeV]$$



for bigger deformation steeper increas of the cumulative integral cross section starting from some energy (see ^{100}Mo)

this starting energy depends on deformation splitting of $S_{E1}(E)$ strength funtion - see next pages

Cumulative integral photoabsorption crossection in the low-energy (Pigmy) region (4 - 13 MeV)

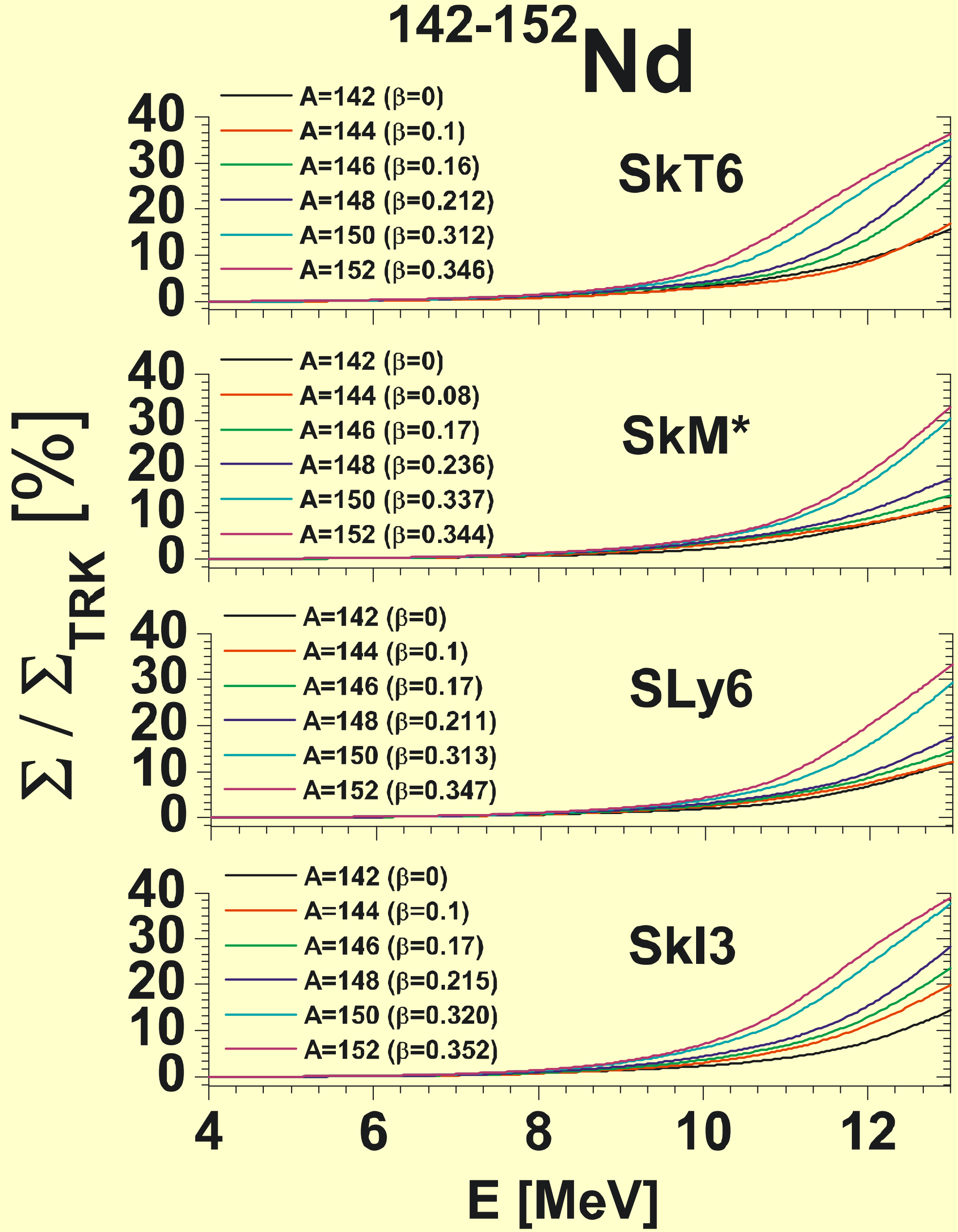
$$\Sigma = \int_4^E dE' \, \sigma_{E1}(E') \quad [mb \; MeV]$$

$$\Sigma_{TRK} \approx 60 \frac{N Z}{A} \quad [mb \; MeV]$$

for bigger deformation steeper increas of the cumulative integral cross section starting from some energy

this starting energy depends on deformation splitting of $S_{E1}(E)$ strength funtion - see next pages

-see also
D.P.Arteaga, E.Khan, P.Ring, PRC 79, 034311 (2009)



E1 excitation strength function

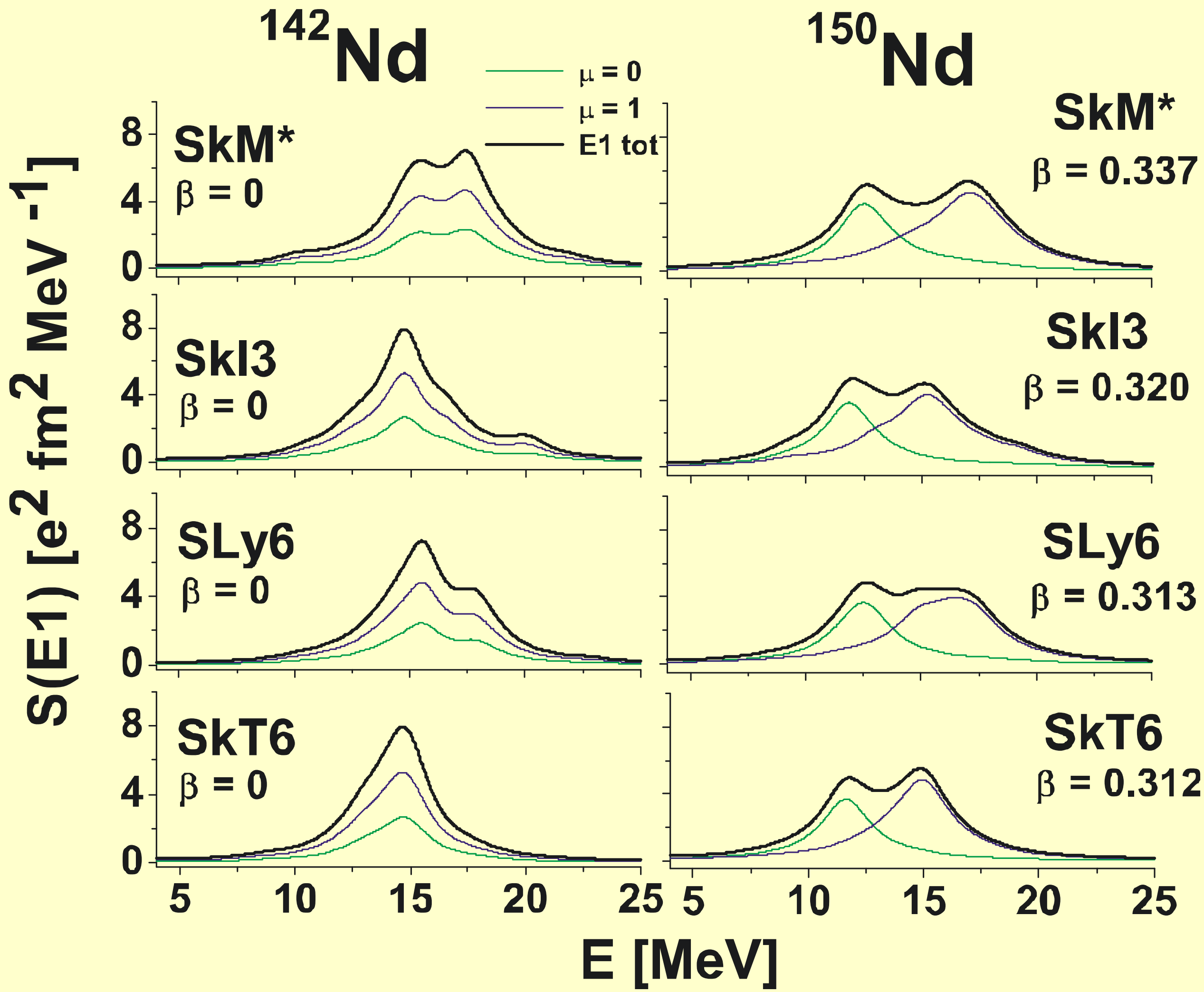
$$S_{E1}(E) = \sum_{\mu=0,1} S_{E1\mu}(E) \quad [e^2\, fm^2 MeV^{-1}]$$

$$S_{E1\mu}(E) = \sum_{\nu} B(E1\mu; |\rangle \to |\nu\rangle)\, \rho_{\Delta}(E - E_{\nu})$$

$$e_{eff}(prot) = \frac{N}{A} e$$

$$e_{eff}(neut) = -\frac{Z}{A} e$$

one can see splitting and broadening of E1 resonance with increasing β



- **exciting operators:**

$$\sum rY_{1\mu}\,;\ \sum r^3 Y_{1\mu}\,;\ \sum r^3 Y_{3\mu}$$

Vorticity

One of the basic questions of all hydrodynamical nuclear models: irrotationality of nuclear matter (with or without whirls?)

irrotationality: $\vec{\nabla} \times \vec{v}(\vec{r}) = 0$

velocity field operator : $\hat{\vec{v}}(\vec{r}) = \dfrac{\hat{\vec{j}}_{nuc}(\vec{r})}{\rho_{nuc}(\vec{r})}$

condition does not guarantee $\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r}) = 0$ because:

$$\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r}) = \vec{\nabla} \times \rho_{nuc}(\vec{r})\, \hat{\vec{v}}(\vec{r}) = \rho_{nuc}(\vec{r})\, \vec{\nabla} \times \hat{\vec{v}}(\vec{r}) + \left(\vec{\nabla}\rho_{nuc}(\vec{r})\right) \times \frac{\hat{\vec{j}}_{nuc}(\vec{r})}{\rho_{nuc}(\vec{r})}$$

$$\Rightarrow \quad \rho_{nuc}(\vec{r})\, \vec{\nabla} \times \hat{\vec{v}}(\vec{r}) = \vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r}) - \left(\vec{\nabla}\rho_{nuc}(\vec{r})\right) \times \hat{\vec{v}}(\vec{r})$$

One can expect that if the nuclear matter is irrotational then:

$$\rho_{nuc}(\vec{r})\, \vec{\nabla} \times \hat{\vec{v}}(\vec{r}) = 0$$

But this is a problem because $\hat{\rho}_{nuc}(\vec{r})$ and $\hat{\vec{j}}_{nuc}(\vec{r})$ are coupled by charge-current conservation:

$$i\,\omega < f \,|\, \hat{\rho}_{nuc}(\vec{r}) \,|\, i > + \vec{\nabla} \cdot < f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, i >$$

The question: how to investigate the irrotationality of nuclear matter in practice?

In papers: D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.C.Caparelli, E.J.V.de Passos, J.Phys.G 25, 537 (1999).
N.Ryezayeva, T.Hartmann, Y.Kalmykov, H.Lenske, P.von Neumann-Cosel, V.Yu.Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).

so called transitional vorticity strength $\nu_\lambda^{(fi)}$ is defined – the idea is folllowing:

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{irrot}(\vec{r}) + \hat{\vec{j}}_{vort}(\vec{r})$$

$$\vec{\nabla}\cdot\hat{\vec{j}}_{vort}(\vec{r}) = 0$$

$$\vec{\nabla}\cdot\hat{\vec{j}}_{irrot}(\vec{r}) = -\frac{i}{\hbar c}[\,\hat{H}\,,\hat{\rho}(\vec{r})\,]$$

vorticity operator: $\hat{\vec{w}}(\vec{r}) \equiv \vec{\nabla}\times\hat{\vec{j}}_{vort}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \vec{\nabla}\times\hat{\vec{j}}_{irrot}(\vec{r})$

or: $< f\,|\,\hat{\vec{w}}(\vec{r})\,|\,i > = \vec{\nabla}\times < f\,|\,\hat{\vec{j}}_{nuc}(\vec{r})\,|\,i > - \vec{\nabla}\times < f\,|\,\hat{\vec{j}}_{irrot}(\vec{r})\,|\,i >$

if nuclear matter is irrotational then all matrix elements of the vorticity operator are zero

for the first sight (see in the previous slide) it seems:

$$\vec{\nabla}\times\hat{\vec{j}}_{irrot}(\vec{r}) = \left(\vec{\nabla}\rho_{nuc}(\vec{r})\right)\times\hat{\vec{v}}(\vec{r})$$

$$\vec{\nabla}\times\hat{\vec{j}}_{vort}(\vec{r}) = \rho_{nuc}(\vec{r})\,\vec{\nabla}\times\hat{\vec{v}}(\vec{r})$$

but it is not so because of the charge-current conservation gives uncertainty what is $\hat{\rho}_{nuc}(\vec{r})$ and what is $\hat{\vec{j}}_{nuc}(\vec{r})$

In the paper D.G.Raventhall, J.Wambach, NPA 475, 468 (1987) decomposition into the spherical vectors $\vec{Y}_{l\lambda\mu}(\vartheta,\varphi)$ is done:

$$< j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i >= \sum_{l\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} \, j^{(fi)}_{l\lambda}(r)\, \vec{Y}^{*}_{l\lambda\mu}(\vartheta,\varphi)$$

$$< j_f m_f \,|\, \hat{\vec{w}}(\vec{r}) \,|\, j_i m_i >= \sum_{l\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} \, w^{(fi)}_{l\lambda}(r)\, \vec{Y}^{*}_{l\lambda\mu}(\vartheta,\varphi)$$

and it was shown (using the charge-current conservation) that

$$< j_f m_f \,|\, \hat{\vec{w}}(\vec{r}) \,|\, j_i m_i >= \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} \, w^{(fi)}_{\lambda\lambda}(r)\, \vec{Y}^{*}_{\lambda\lambda\mu}(\vartheta,\varphi)$$

with

$$w^{(fi)}_{\lambda\lambda}(r) = \sqrt{\frac{2\lambda+1}{\lambda}} \left(\frac{d}{dr} + \frac{\lambda+2}{r} \right) j^{(fi)}_{\lambda\,\lambda+1}(r)$$

- all information about the transitional vorticity $\vec{w}^{(fi)}(\vec{r})$ is given by the radial transitional component $j^{(fi)}_{\lambda\,\lambda+1}(r)$ of the nuclear charge current
- it was also shown that $\vec{\nabla} \times \hat{\vec{j}}_{vort}(\vec{r}) \;\leftrightarrow\; \frac{1}{\lambda} \rho_{nuc}(\vec{r})\, \vec{\nabla} \times \hat{\vec{v}}(\vec{r})$

In papers:

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.C.Caparelli, E.J.V.de Passos, J.Phys.G 25, 537 (1999).
N.Ryezayeva, T.Hartmann, Y.Kalmykov, H.Lenske, P.von Neumann-Cosel, V.Yu.Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).

so called transition vorticity strength $\nu_{\lambda}^{(fi)}$:

$$\nu_{\lambda}^{(fi)} = \int_0^{\infty} r^{\lambda+4}\, w_{\lambda\lambda}^{(fi)}(r)\, dr$$

was introduced as a measure of the irrotationality of the nuclear matter (usually $\lambda = 1$).

It was shown that the vorticity strength is significant for the transitions $|i> = |RPA> \to |f>$ from the ground state to states $|f>$ in the Pigmy region and that these states have a toroidal character (for some lighter spherical nuclei).

we introduced another quantity as a measure of the irrotationality - vorticity multipole operator

vorticity multipole operator is directly connected with the long-wave decomposition of the standard electric multipole operator:

$$\hat{M}_{E\lambda\mu}(k) = \frac{(2\lambda+1)!!}{c\,k^{\lambda+1}(\lambda+1)}\sqrt{\lambda(\lambda+1)}\int d^3r\,[j_\lambda(kr)\,\vec{Y}_{\lambda\lambda\mu}(\vartheta,\varphi)]\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})] =$$

$$= \left\{ \text{using Bessel function decomposition} \quad j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}\left[1 + \frac{-\frac{k^2r^2}{2}}{2\lambda+3} + \cdots\right] \right\} =$$

$$= \hat{M}_{E\lambda\mu}(k=0) + k\,\hat{M}_{tor\,\lambda\mu}(k=0) + \cdots$$

toroidal multipole operator see e.g.

D.Vretenar, N.Paar, P.Ring, T.Niksic, PRC 65, 021301 (2002)

S.F.Semenko, Yad.Fiz. 34, 639 (1981)

(nonstandard normalization of el.mg. multipoles)

where $\hbar kc$ is the transition energy and

$$\hat{M}_{E\lambda\mu}(k=0) = \int d^3r\,\hat{\rho}_{nuc}(\vec{r})\,r^\lambda\,Y_{\lambda\mu}(\vartheta,\varphi)$$

$$\hat{M}_{tor\,\lambda\mu}(k=0) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d^3r\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}\left[\vec{Y}_{\lambda-1\lambda\mu}(\vartheta,\varphi) - \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda+1\lambda\mu}(\vartheta,\varphi)\right]$$

Vorticity multipole operator is obtained from $\hat{M}_{E\lambda\mu}(k)$ **(see exp. ✡)**
by the following substitution:

$$\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) \Longrightarrow \hat{\vec{w}}(\vec{r}) \equiv \vec{\nabla}\times\hat{\vec{j}}_{vort}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \vec{\nabla}\times\hat{\vec{j}}_{irrot}(\vec{r}) =$$

in accordance with D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$= \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \frac{1}{\lambda}\left(\vec{\nabla}\rho_{nuc}(\vec{r})\right)\times\hat{\vec{v}}(\vec{r})$$

Then

$$\hat{M}_{vor\,\lambda\mu}(k) = \frac{(2\lambda+1)!!}{c\,k^{\lambda+1}(\lambda+1)}\sqrt{\lambda(\lambda+1)}\int d^3r\,[j_\lambda(kr)\,\vec{Y}_{\lambda\lambda\mu}(\vartheta,\varphi)]\cdot$$

$$\cdot\left[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \frac{1}{\lambda}\left(\vec{\nabla}\rho_{nuc}(\vec{r})\right)\times\hat{\vec{v}}(\vec{r})\right] =$$

$$= \left\{\text{Bessel function decomposition}\right\} = k\,\hat{M}_{vor\,\lambda\mu}(k=0) + \cdots$$

There is no zero order term in the $k-$decomposition

with long-wave limit of the vorticity multipole operator:

$$\hat{M}_{vor\,\lambda\mu}(k=0) = -\frac{i}{c}\frac{1}{2\lambda+3}\sqrt{\frac{2\lambda+1}{\lambda+1}}\int d^3r\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot\vec{Y}_{\lambda+1\lambda\mu}(\vartheta,\varphi)\,r^{\lambda+1}$$

The nonzero value of all matrix elements of all vorticity multipole operators can serve as a measure of the irrotationality of the nuclear matter. We restrict ourselves for $\lambda = 1$ and we calculate the strength function of $\hat{M}_{vor\,1\mu}(k=0)$:

$$S(vor\,1;E) = \sum_{\mu=0,\pm1} S(vor\,1\mu) =$$

$$= \sum_{\mu\nu} |< RPA \,|\, [\, O_\nu, \hat{M}_{vor\,1\mu}(k=0)\,] \,|\, RPA >|^2 \, \delta(E-\omega_\nu)$$

Dipole vorticity strength function can be compared with the dipole toroidal strength function:

$$S(tor\,1;E) = \sum_{\mu=0,\pm1} S(tor\,1\mu) =$$

$$= \sum_{\mu\nu} |< RPA \,|\, [\, O_\nu, \hat{M}_{tor\,1\mu}(k=0)\,] \,|\, RPA >|^2 \, \delta(E-\omega_\nu)$$

where dipole toroidal operator (involving the corrections to the C.o M. motion) is:

$$\hat{M}_{tor\,1\mu}(k=0) = -\frac{i}{2c}\sqrt{\frac{1}{3}}\int d^3r\, \hat{\vec{j}}_{nuc}(\vec{r})\cdot$$

$$\left\{ r^2\left[\vec{Y}_{01\mu}(\vartheta,\varphi) - \frac{\sqrt{2}}{5}\vec{Y}_{21\mu}(\vartheta,\varphi)\right] - \langle r^2\rangle \vec{Y}_{01\mu}(\vartheta,\varphi)\right\}$$

correction to C.o.M motion

Dipole vorticity strength function can be also compared with the squeezed dipole electric (or isoscalar E1) strength function:

$$S(sq\;E1;E) = \sum_{\mu=0,\pm1} S(sq\;E1\mu) =$$

$$= \sum_{\mu\nu} |\langle RPA\,|\,[\,O_\nu, \hat{M}_{sq\,E1\mu}(k=0)\,]\,|\,RPA\rangle|^2\, \delta(E-\omega_\nu)$$

with the squeezed dipole E1 transition operator:

correction to C.o.M motion

$$\hat{M}_{sq\,E1\mu}(k=0) = \int d^3r\, \hat{\rho}(\vec{r})\left[r^3 - \frac{5}{3}\langle r^2\rangle r\right] Y_{1\mu}(\vartheta,\varphi)$$

Connection between squeezed dipole E1 operator and the dipole toroidal operator is discussed in J.Kvasil, N.Lo Iudice, Ch.Stoyanov, P.Alexa, J.Phys G 29, 753 (2003)

Nuclear charge density operator:

$$\hat{\rho}_{nuc}(\vec{r}) = e \sum_{\tau=n,p} e_{eff}^{(\tau)} \sum_{ij\in\tau} \psi_i^+(\vec{r})\,\psi_j(\vec{r})\,a_i^+ a_j$$

Nuclear charge current operator consists from convectional and magnetization parts:

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) + \hat{\vec{j}}_{mag}(\vec{r})$$

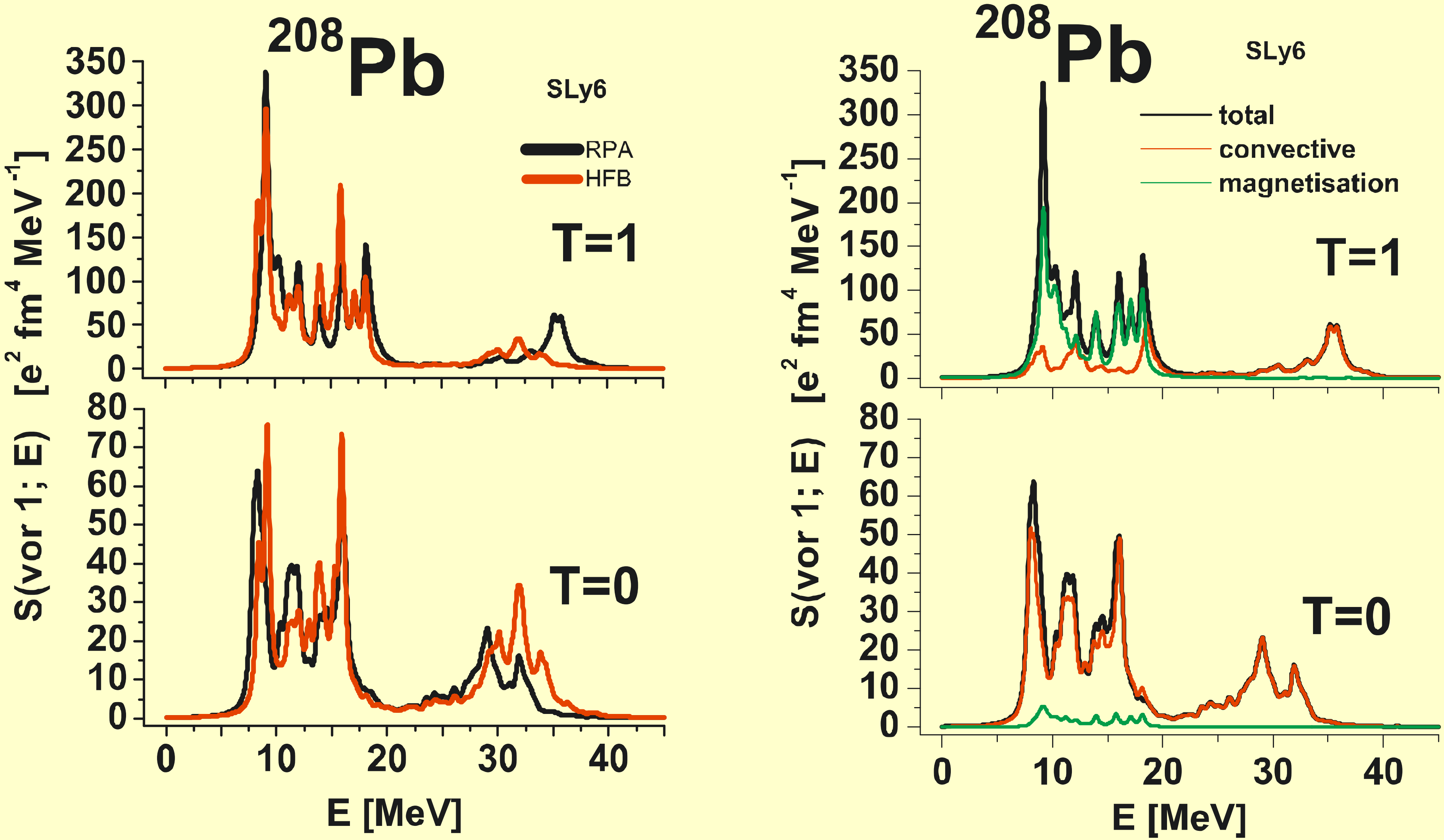
$$\hat{\vec{j}}_{con}(\vec{r}) = \frac{i}{2}\frac{e\hbar}{m} \sum_{\tau=n,p} e_{eff}^{(\tau)} \sum_{ij\in\tau} \left[\left(\vec{\nabla}\psi_i^+(\vec{r})\right) \psi_j(\vec{r}) - \psi_i^+(\vec{r}) \left(\vec{\nabla}\psi_j(\vec{r})\right) \right] a_i^+ a_j$$

$$\hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{2m} \sum_{\tau=n,p} g_{eff}^{(\tau)} \sum_{ij\in\tau} \left[\vec{\nabla}\times\left(\psi_i^+(\vec{r})\ \vec{\sigma}\ \psi_j(\vec{r})\right) \right] a_i^+ a_j$$

where effective charges and gyromagnetic ratios depend on the process of excitation (see M.N.Harakeh, A.van der Woude, Giant Resonances, Clarendon 2001)

	el.mag.	isoscalar T=0	isovector T=1
$g_{spin}^{(p)} = \varsigma\, 5.58$	$e_{eff}^{(p)} = 1 \quad e_{eff}^{(n)} = 0$	$e_{eff}^{(p)} = 1 \quad e_{eff}^{(n)} = 1$	$e_{eff}^{(p)} = 1 \quad e_{eff}^{(n)} = -1$
$g_{spin}^{(p)} = -\varsigma\, 3.82$	$g_{eff}^{(p)} = g_{spin}^{(p)}$	$g_{eff}^{(p)} = (g_{spin}^{(p)} + g_{spin}^{(n)})/2$	$g_{eff}^{(p)} = (g_{spin}^{(p)} - g_{spin}^{(n)})/2$
$\varsigma \approx 0.7$	$g_{eff}^{(n)} = g_{spin}^{(n)}$	$g_{eff}^{(n)} = (g_{spin}^{(p)} + g_{spin}^{(n)})/2$	$g_{eff}^{(n)} = -(g_{spin}^{(p)} - g_{spin}^{(n)})/2$

- isoscalar vorticity strength is mainly formed by convective part of the nuclear charge current
- isovector vorticity strength is mainly formed by magnetization part of the nuclear charge current



vorticity – exc. operators:

$$\int d^3r\; \hat{\vec{j}}_{con}(\vec{r})\cdot r^2\vec{Y}_{21\mu} \qquad \int d^3r\; \hat{\rho}(\vec{r})\; r^3 Y_{1\mu}$$

$$\int d^3r\; \hat{\vec{j}}_{mag}(\vec{r})\cdot r^2\vec{Y}_{21\mu}$$

toroidal– exc. operators:

$$\int d^3r\; \hat{\vec{j}}_{con}(\vec{r})\cdot r^2\,[\vec{Y}_{01\mu} - \sqrt{\frac{1}{2}}\,\frac{2}{5}\vec{Y}_{21\mu}]$$

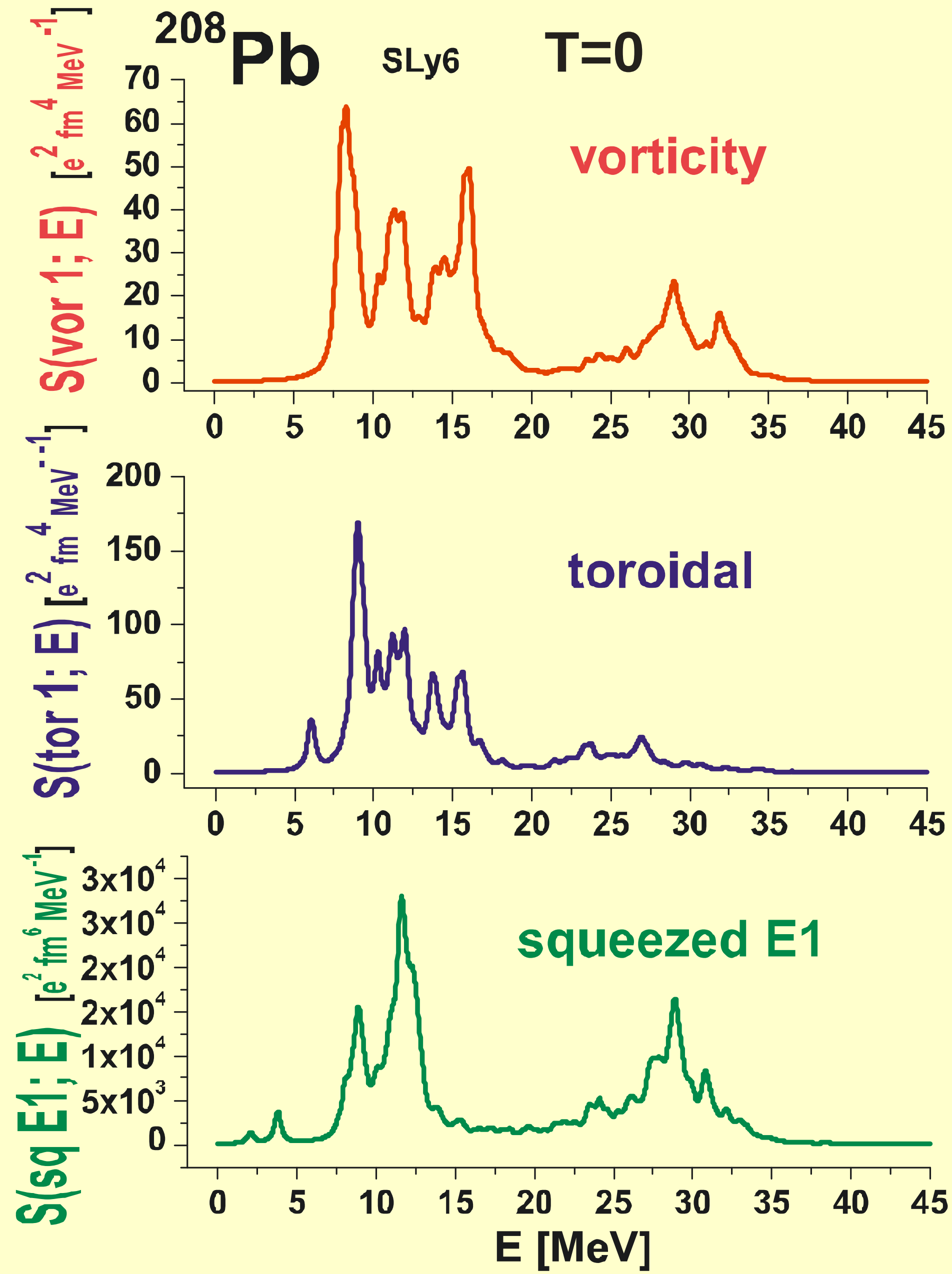
$$\int d^3r\; \hat{\vec{j}}_{mag}(\vec{r})\cdot r^2\,[\vec{Y}_{01\mu} - \sqrt{\frac{1}{2}}\,\frac{2}{5}\vec{Y}_{21\mu}]$$

$$\int d^3r\; \hat{\rho}(\vec{r})\; r^3 Y_{1\mu}$$

squeezed E1– exc. operators:

$$\int d^3r\; \hat{\vec{j}}_{con}(\vec{r})\cdot r^2\,[\vec{Y}_{01\mu} - \sqrt{\frac{1}{2}}\,\frac{2}{5}\vec{Y}_{21\mu}]$$

$$\int d^3r\; \hat{\rho}(\vec{r})\; r^3 Y_{1\mu}$$



Conclusions

- SRPA – effective method for the investigation of excited states in heavy nuclei

- different Skyrme parametrisations (SkI3, SkM*, SkT6, SLy6) give very similar and good agreement with experimental photoabsorption cross section (not so for M1 giant resonance)

- for bigger deformation steeper increase of the cumulative integral photoabsorption cross section with the increasing excitation energy is observed for energies above the particle emission threshold. Below this threshold this increase is not so conclusive

- significant vorticity dipole strength is observed in the excitation energy intervals (for ^{208}Pb):

$$7\ MeV < E < 20\ MeV$$

$$27\ MeV < E < 37\ MeV$$

in these energy intervals one can expect a significant irrotationality of the nuclear matter in positive parity excited states

- isoscalar dipole vorticity strength is mainly formed by the convective charge current while the isovector dipole vorticity strength (low energy part) is mainly caused by the magnetization charge current