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Radiative corrections in Dalitz decays of π^0 , η and η' mesons

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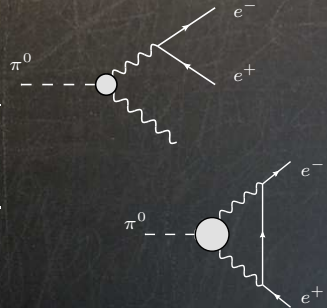
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Decay modes of neutral pion:

Process	Branching ratio
$\pi^0 \rightarrow \gamma\gamma$	$(98.823 \pm 0.034) \%$
$\pi^0 \rightarrow e^+e^-\gamma$	$(1.174 \pm 0.035) \%$
$\pi^0 \rightarrow e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \rightarrow e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



Rare decay $\pi^0 \rightarrow e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to **Drell, NC (1959)**
- suppressed in comparison to the decay $\pi^0 \rightarrow \gamma\gamma$ by a factor of $2(\alpha m_e/M\pi)^2$
 - \hookrightarrow one-loop structure + helicity suppression
 - \hookrightarrow may be sensitive to possible effects of new physics

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KTeV-E799-II experiment at Fermilab (*Abouzaid et al., PRD 75 (2007)*)
 \hookrightarrow **precise** measurements of branching ratio $\pi^0 \rightarrow e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B^{\text{KTeV}}(\pi^0 \rightarrow e^+e^-(\gamma), x_D > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
- extrapolate full radiative tail beyond $x > 0.95$ (*Bergström, Z.Ph.C 20 (1983)*)
- scale the result back by the overall radiative corrections

\hookrightarrow **final result** for lowest order (no final state radiation)

$$B_{\text{KTeV}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

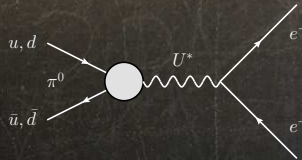
Comparison with SM prediction (*Dorokhov and Ivanov, PRD 75 (2007)*)

$$B_{\text{SM}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

\hookrightarrow interpreted as **3.3 σ discrepancy** between theory and experiment

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- very fashionable to ascribe eventual discrepancies to effects of new physics

BUT

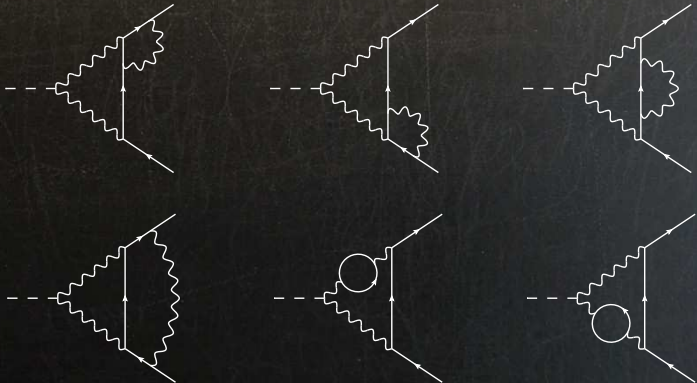
- first, look for more conventional solution (i.e. within SM)

- ↪ radiative corrections (usually very important)
- ↪ form-factor modeling

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- calculated by *Vaško and Novotný, JHEP 1110 (2011)*



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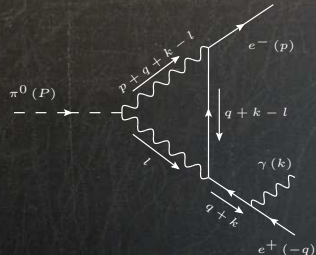
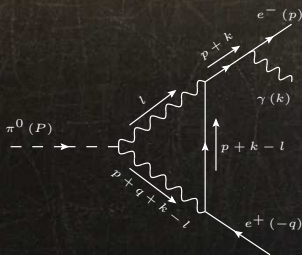
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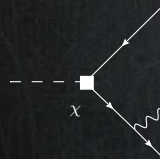
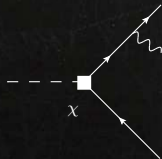
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- compensation of **IR** divergence in 2-loop contributions
 \hookrightarrow *TH, Kampf and Novotný, EPJC 74 (2014)*



- contain **UV** subdivergences \rightarrow counter-term tree diagrams with couplig χ



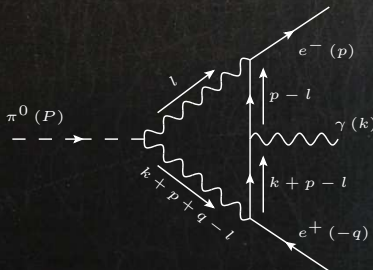
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Do not forget the third, **box** diagram, necessary to satisfy the **Ward identities**

$$\mathcal{M}_{(\lambda)} = \varepsilon_{(\lambda)}^{*\rho}(k) \mathcal{M}_{\rho}^{\text{BS}} \longrightarrow k^{\rho} \mathcal{M}_{\rho}^{\text{BS}} = 0$$

- **finite** contribution to bremsstrahlung amplitude



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Size of the radiative corrections (**newly** calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2) \%$$

- can be thought as model-independent
- differs **significantly** from previous **approximate** calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8 \%$

Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3 \%$

- original KTeV vs. SM discrepancy reduced to the 2σ level or less
- contact interaction coupling finite part set to

$$\chi_{\text{LMD}}^{(r)}(M_\rho) = 2.2 \pm 0.9$$

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Quantity **really** measured by KTeV

$$\left. \frac{\Gamma(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma(\gamma), x > 0.2319)} \right|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

↪ Dalitz decay comes into play

- **second** most important decay channel of a neutral pion
 ↪ branching ratio $(1.174 \pm 0.035) \%$
- first studied by **Richard H. Dalitz**, PPSA 64 (1951), whose name it carries
- experimental data of this process provide the information about **singly** virtual pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$
 ↪ in particular about its **slope** parameter a_π

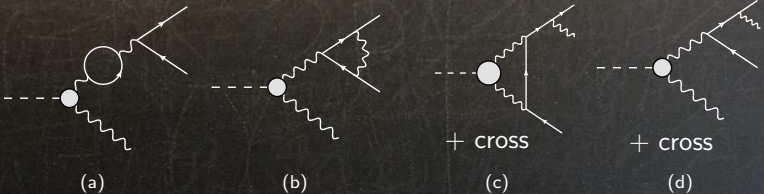
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- radiative corrections to the **total** decay rate of the Dalitz decay
 ⇨ first addressed (numerically) by *Joseph, NC 16 (1960)*
- pioneering study of corrections to the **differential** decay rate
 ⇨ *Lautrup and Smith, PRD 3 (1971)*
 ⇨ soft-photon approximation
- extended by *Mikaelian and Smith, PRD 5 (1972)*
 ⇨ hard-photon corrections
 ⇨ **whole** range of bremsstrahlung photon energy
 ⇨ table of values

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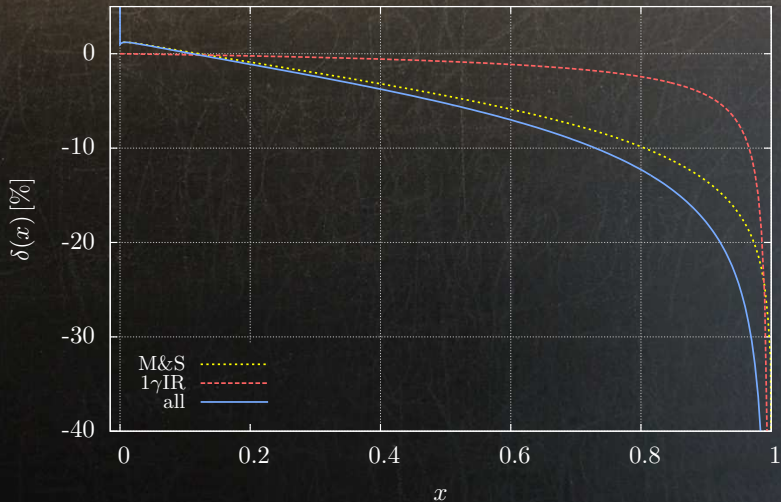
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- new calculations motivated by needs of NA48/NA62 experiments at CERN
 \hookrightarrow measure the slope a_π of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$: *Lazzeroni et al., PLB 768 (2017)*
- unlike before **no approximation** was used
 \hookrightarrow can be used also for related decays $\eta \rightarrow \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
 \hookrightarrow propagated into **MC generator** of NA62 experiment
- *TH, Kampf and Novotný, PRD 92 (2015)*

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Precise knowledge of $R \equiv \frac{\Gamma(\pi^0 \rightarrow e^+e^-\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)}$

↪ for small **slope** and up to **NLO radiative corrections**

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x, y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] dx dy$$

Conservative estimate for uncertainty (a_π , NNLO)

$$R = 1.1978(6) \%$$

↪ **chosen** $a_\pi = 3.55(70) \%$, covers

$$a_\pi^{\text{VMD}} = 3.0 \%, \quad a_\pi^{\text{PDG}} = 3.35(31) \%, \quad a_\pi^{\text{NA62}} = 3.68(57) \%$$

Constraint: $1 \simeq \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) + \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \rightarrow e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.8131(6) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) = 1.1836(6) \%$$

TH, Goudzovski and Kampf, to appear on arXiv soon

PDG

$$R = 1.188(35) \%, \quad \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.823(34) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma) = 1.174(35) \%$$

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$\eta^{(\prime)}$ Dalitz decays

- small branching ratios
 \hookrightarrow hadronic decay modes are open
- access to electromagnetic transition form factors
 $\hookrightarrow \eta^{(\prime)}$ -meson structure
 \hookrightarrow valuable input for other quantities and e.g. $g - 2$ of a muon
 \hookrightarrow radiative corrections crucial to **extract** relevant information from data

naive rad. corrections for $\eta \rightarrow e^+e^-\gamma$: *Mikaelian and Smith, PRD 5 2890 (1972)*

- numerical values correspond to simple change $M_{\pi^0} \rightarrow M_\eta$
 $\hookrightarrow \pi^0$ case: *Mikaelian and Smith, PRD 5 1763 (1972)*
- fully inclusive radiative corrections
 \hookrightarrow **no** momentum or angular cuts on the bremsstrahlung photon applied

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The $\eta^{(\prime)}$ case compared to π^0

- larger rest mass

↪ M_η above muon-pair threshold: $M_\eta > 2m_\mu$

↪ $M_{\eta'}$ above lowest-lying resonances: $M_{\eta'} > M_\rho, M_\omega$

↪ sensitive to the **widths** of resonances

↪ ω narrow, ρ **broad** resonance in $\pi\pi$ scattering

- strange-flavor content

↪ quark-flavor basis

Feldmann et al., PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2} [\bar{u}\gamma_5 u + \bar{d}\gamma_5 d], \quad j^s \equiv \frac{i}{\sqrt{2}} [\bar{s}\gamma_5 s]$$

- η - η' **mixing**: $\langle 0 | j^A | \eta^B \rangle = B_0 F_\pi f_A \delta^{AB}$, $\langle \eta^A | \eta^B \rangle = \delta^{AB}$, $A, B \in \{\ell, s\}$

$$|\eta\rangle = \cos \phi |\eta^\ell\rangle - \sin \phi |\eta^s\rangle$$

$$|\eta'\rangle = \sin \phi |\eta^\ell\rangle + \cos \phi |\eta^s\rangle$$

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Full set of NLO QED radiative corrections:

TH, Kampf, Leupold and Novotný, PRD 97 (2018)

- compared to previous approach:
 - ↪ muon loops + **hadronic** VP
 - ↪ **1 γ IR** at one-loop level
 - ↪ **form-factor** effects (also in BS)
 - ↪ higher orders in the final-state lepton mass **not** neglected

- general framework: **three** additional processes
 - ↪ also muon decay modes

η case: **most** of the ingredients in *TH, Kampf and Novotný, PRD 92 (2015)*

η' case: real challenge

- ↪ resulting framework also **applicable** to the π^0 case (numerically compatible)
 - ↪ overkill (correction to the correction of order 1%)

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Photon self-energy in the form $\Pi(s) = \Pi_L(s) + \Pi_H(s)$

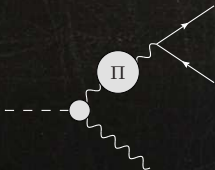
- lepton loops (electrons and as well **muons**)

$$\Pi_L(M_P^2 x) = \frac{\alpha}{\pi} \sum_{\ell'=e,\mu} \left\{ \frac{8}{9} - \frac{\beta_{\ell'}^2}{3} + \left(1 - \frac{\beta_{\ell'}^2}{3} \right) \frac{\beta_{\ell'}}{2} \log [-\gamma_{\ell'} + i\epsilon] \right\}$$

- **hadronic** contribution

\hookrightarrow *Jegerlehner, Z.Ph.C 32 (1986)*

$$\Pi_H(s) = -\frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_H(s') ds'}{s - s' + i\epsilon}$$



$$\delta^{\text{virt}}(x, y) = \frac{1}{|1 + \Pi(M_P^2 x)|^2} - 1 + 2 \text{Re} \left\{ F_1(x) + \frac{2F_2(x)}{1 + y^2 + \frac{\nu^2}{x}} \right\}$$

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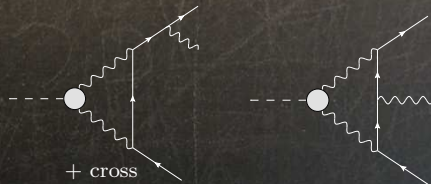
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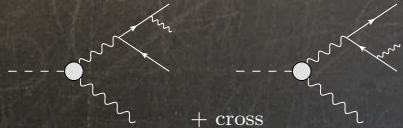
1 γ IR contribution at one-loop level

- beyond effective approach
- we don't expect substantial model dependence \leftrightarrow VMD-inspired model \leftrightarrow strange-flavor content and η - η' mixing

$$e^2 \mathcal{F}_{\eta\gamma^*\gamma^*}^{\text{VMD}}(p^2, q^2) = -\frac{N_c}{8\pi^2 F_\pi} \frac{2e^2}{3} \times \left[\frac{5}{3} \frac{\cos \phi}{f_\ell} \frac{M_{\omega/\rho}^4}{(p^2 - M_{\omega/\rho}^2)(q^2 - M_{\omega/\rho}^2)} - \frac{\sqrt{2}}{3} \frac{\sin \phi}{f_s} \frac{M_\phi^4}{(p^2 - M_\phi^2)(q^2 - M_\phi^2)} \right]$$

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- slope not negligible
- for η : expansion in slope a would be **still** (somewhat) suitable

$$\mathcal{F}((p_\gamma + p_{e^+} + p_{e^-})^2) \simeq \mathcal{F}(M_P^2 x) \left[1 + a \frac{2p_\gamma \cdot (p_{e^+} + p_{e^-})}{M_P^2} \right]$$

- for η' : such an expansion **not applicable** anymore
- \hookrightarrow BS necessarily depends on the form-factor model

sensitivity to width of ρ meson \hookrightarrow recent **dispersive** calculations used
Hanhart et al., EPJC 73 (2013), EPJC 77 (2017)

Källén–Lehmann spectral representation \rightarrow common spectral density function

$$\frac{\mathcal{F}(q^2)}{\mathcal{F}(0)} \simeq 1 + q^2 \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathcal{A}(s) ds}{q^2 - s + i\epsilon}$$

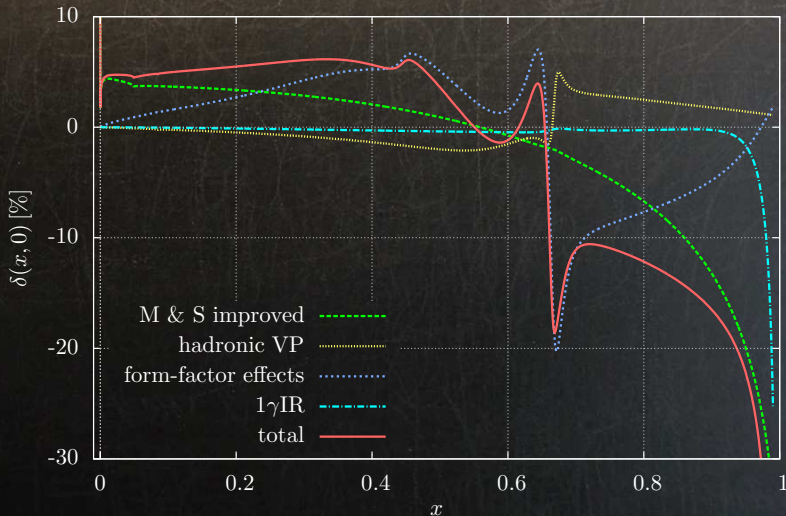
$$\mathcal{A}(s) = w_\omega \mathcal{A}_\omega(s) + w_\phi \mathcal{A}_\phi(s) - \frac{\kappa}{96\pi^2 F_\pi^2} \left[1 - \frac{4m_\pi^2}{s} \right]^{3/2} P(s) R(s) |\Omega(s)|^2$$

Radiative corrections to $\eta' \rightarrow e^+e^-\gamma$ decays

The overall NLO correction $\delta(x, 0)$ in comparison to its constituents

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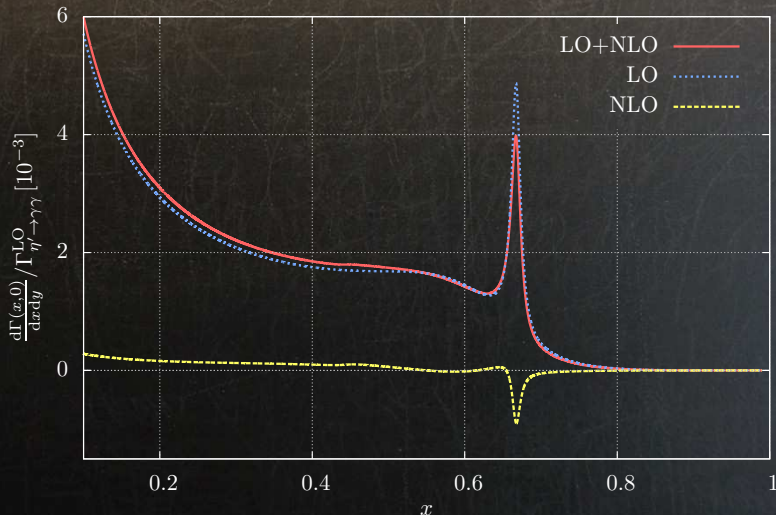


Radiative corrections to $\eta' \rightarrow e^+e^-\gamma$ decays

The two-fold differential decay width $d\Gamma(x,0)$ at NLO

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All NLO QED radiative corrections for discussed processes are now available
 \hookrightarrow can be taken into account in **future** experimental analyses

- $\pi^0 \rightarrow e^+e^-$

Vaško and Novotný, JHEP 1110 (2011)

TH, Kampf and Novotný, EPJC 74 (2014)

- $\pi^0 \rightarrow e^+e^-\gamma$

TH, Kampf and Novotný, PRD 92 (2015)

\hookrightarrow **precise** knowledge of R

- $\eta^{(\prime)} \rightarrow \ell^+\ell^-\gamma$

TH, Kampf, Leupold and Novotný, PRD 97 (2018)

Ancillary files available together with the papers

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Thank you for listening!