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Corrections Beyond the Leading Order in Processes $\pi^0 \to e^+e^-$ and $\pi^0 \to e^+e^-\gamma$

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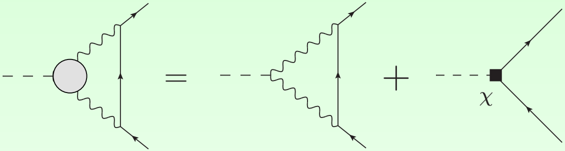


- rare decay $\pi^0 \to e^+e^-$
- more precise measurements of the branching ratio performed by KTeV experiment at Fermilab (*Abouzaid et al.*, 2007 [1])

$$B(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- theoretical prediction according to the SM seams not to correspond to the measurements by 3.3 σ (*Dorokhov and Ivanov*, 2007 [2])
- this discrepancy has not been satisfactorily explained yet

- today, it is very fashionable to ascribe the eventual discrepancies to the effects of a new physics

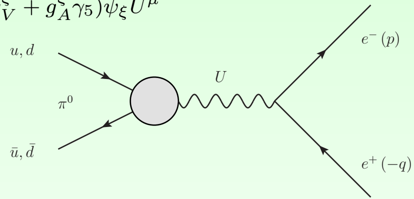
- miscellaneous theoretical models can be proposed and excluded due to various observations
- 511 keV signal coming from the center of the galaxy

  $\rightarrow$ corresponds to the annihilation of a great amount of e^+e^- pairs
  $\rightarrow$ cannot be explained by presently known astrophysical sources
- consistent with the scalar dark matter model

  $\rightarrow$ new light neutral gauge boson
- one of the possible contributions of a new physics to the process $\pi^0 \rightarrow e^+e^-$

In this model (*Boehm and Fayet*, 2004 [3]), we take simply

$$\mathcal{L}_{\text{int}}^{U,f} \stackrel{\text{eff}}{=} \sum_{\xi=u,d,e} \bar{\psi}_\xi \gamma_\mu (g_V^\xi + g_A^\xi \gamma_5) \psi_\xi U^\mu$$



To obtain the relevant informations, we assume (not excluded by observations)

- $M_U \simeq 10$ MeV (*Fayet*, 2006 [4])
- $g_A^e \simeq (g_A^d - g_A^u)$

The final estimate of the desired coupling in order to solve the discrepancy

$$|g_A^e| \simeq 2 \times 10^{-4}$$

- all in all, it is always necessary to check if there is really no more conventional solution (i.e. within SM)
- radiative corrections (real or virtual) are usually very important

Leading order

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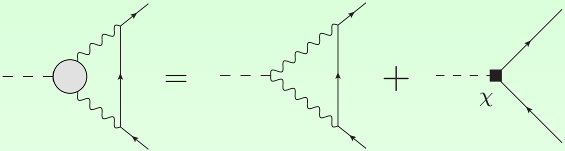


- l.h.s.: LO contribution in the QED expansion
- r.h.s.: its representation as the LO of the χPT

$$^{\text{tree}}\mathcal{L}_{\text{WZW}}^{\pi^0\to\gamma\gamma} = -\frac{e^2}{2}\frac{1}{4\pi^2 F}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma)\pi^0 \quad \longrightarrow \quad F_{\pi^0\gamma^*\gamma^*}^{\text{LO}} = \frac{1}{4\pi^2 F}$$

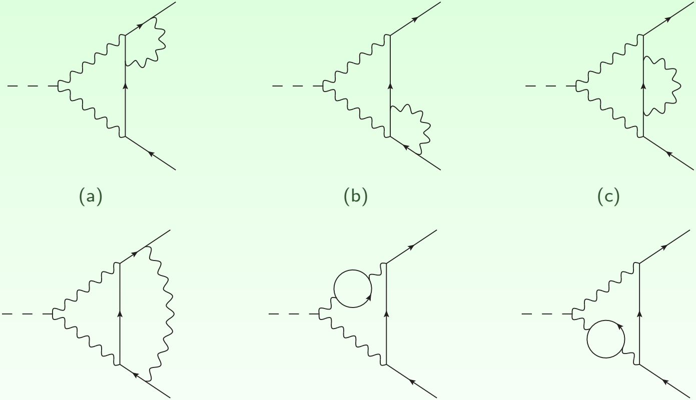
Local counter-term chiral Lagrangian à la *Savage et al.*, 1992 [5]

$$\mathcal{L}_{\pi l\bar{l}} = \frac{3i}{2}\left(\frac{\alpha}{4\pi}\right)^2 (\bar{l}\gamma^\mu\gamma_5 l) \times$$
$$\times\left\{\chi_1 \mathrm{Tr}[Q^2\partial_\mu U U^\dagger - Q^2\partial_\mu U^\dagger U] + \chi_2 \mathrm{Tr}[Q\partial_\mu U Q U^\dagger - Q\partial_\mu U^\dagger Q U]\right\}$$

$$\chi = -\frac{(\chi_1+\chi_2)}{4} \overset{\text{LO}}{=} \frac{3}{2}\left(\frac{1}{\epsilon} - \gamma_{\text{E}} + \log 4\pi\right) + \chi^{(r)}(\mu)$$

Two-loop virtual radiative corrections
Diagrams

Contents



(a) (b) (c)

(d) (e) (f)

- calculated by *Vaško and Novotný*, 2011 [6]
- the result can be expressed in terms of the correction factor $\delta(x^{\mathrm{cut}})$ defined as

$$\Gamma^{\mathrm{NLO}}\left(\pi^0 \to e^+e^-(\gamma),\, x > x^{\mathrm{cut}}\right) \equiv \delta\left(x^{\mathrm{cut}}\right)\Gamma^{\mathrm{LO}}\left(\pi^0 \to e^+e^-\right)$$

Size of the two-loop correction

$$\delta^{(\text{2-loop})}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}_{\text{soft}}(0.95) = (-5.8 \pm 0.2)\,\%$$

- differs significantly from the previous approximate calculations

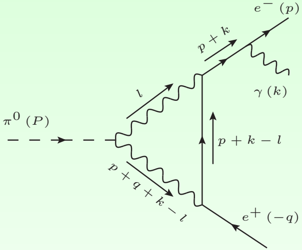
$\to$ *Bergström*, 1983 [7]: $\delta^{(\text{2-loop})}(0.95) = -13.8\,\%$
$\to$ *Dorokhov et al.*, 2008 [8]: $\delta^{(\text{2-loop})}(0.95) = -13.3\,\%$

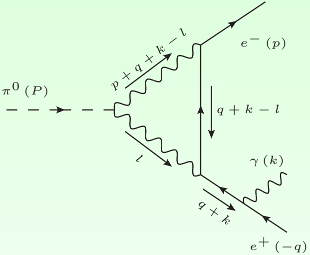
- the original discrepancy has been reduced to the $2\,\sigma$ level

Bremsstrahlung
photon emission from the outer fermion line

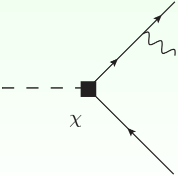
Contents

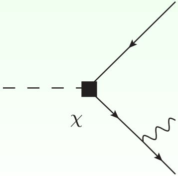
- compensation of IR divergence in 2-loop contributions





- contain UV subdivergences $\rightarrow$ counter-term tree diagrams with couplig χ

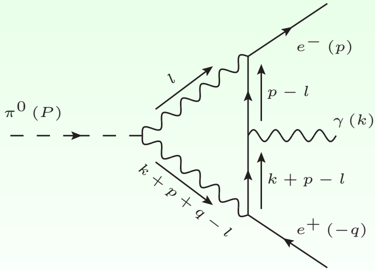




Do not forget the third, box diagram, necessary to fulfill the Ward identities

$$\mathcal{M}_{(\lambda)} = \varepsilon^{*\rho}_{(\lambda)}(k)\mathcal{M}^{\mathsf{BS}}_{\rho} \quad \longrightarrow \quad k^\rho \mathcal{M}^{\mathsf{BS}}_{\rho} = 0$$

- finite contribution to the bremsstrahlung amplitude



$$
\begin{aligned}
i\mathcal{M}_{(\lambda)}(p,q,k) = \frac{ie^5}{8\pi^2 F} \times \epsilon^{*\rho}_{(\lambda)}(k) \\
\times \Big\{ P(x,y)\left[(k\cdot p)q_\rho - (k\cdot q)p_\rho\right]\left[\bar{u}(p,m)\gamma_5 v(q,m)\right] \\
+A(x,y)\Big[\bar{u}(p,m)\left[\gamma_\rho(k\cdot p) - p_\rho(k\cdot\gamma)\right]\gamma_5 v(q,m)\Big] \\
-A(x,-y)\Big[\bar{u}(p,m)\left[\gamma_\rho(k\cdot q) - q_\rho(k\cdot\gamma)\right]\gamma_5 v(q,m)\Big] \\
+T(x,y)[\bar{u}(p,m)\gamma_\rho \not{k}\gamma_5 v(q,m)]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{|\mathcal{M}^{\mathrm{BS}}(x,y)|^2} \equiv \sum_{\text{polarizations } \lambda} \left|\mathcal{M}_{(\lambda)}(p,q,k)\right|^2 = \\
= \frac{16\pi\alpha^5}{F^2}\frac{M^4(1-x)^2}{8}\Big\{M^2\Big[x(1-y^2)-\nu^2\Big]\Big[xM^2|P|^2 \\
+ 2\nu M\,\mathrm{Re}\left\{P^*\left[A(x,y)+A(x,-y)\right]\right\} - 4\,\mathrm{Re}\left\{P^*T\right\}\Big] \\
+ 2M^2(x-\nu^2)(1-y)^2|A(x,y)|^2 + (y\to -y) \\
- 8\nu M y(1-y)\,\mathrm{Re}\left\{A(x,y)T^*\right\} + (y\to -y) \\
- 4\nu^2M^2y^2\,\mathrm{Re}\left\{A(x,y)A(x,-y)^*\right\} + 8(1-y^2)|T|^2\Big\}
\end{aligned}
$$

- general formula in terms of Dalitz kinematic variables x and y

$$\mathrm{d}\Gamma(x,y) = \frac{M}{(8\pi)^3}\overline{|\mathcal{M}_{\mathrm{fi}}(x,y)|^2}(1-x)\mathrm{d}x\mathrm{d}y$$

- we also subtract the soft-photon approximation term already included in two-loop calculation done by *Vaško and Novotný,* 2011 [6]

$$\mathrm{d}\Gamma^{\mathrm{BS}}_{\mathrm{diff}}(x,y) = \mathrm{d}\Gamma^{\mathrm{BS}}(x,y) - \mathrm{d}\Gamma^{\mathrm{BS}}_{\mathrm{soft}}(x,y)$$

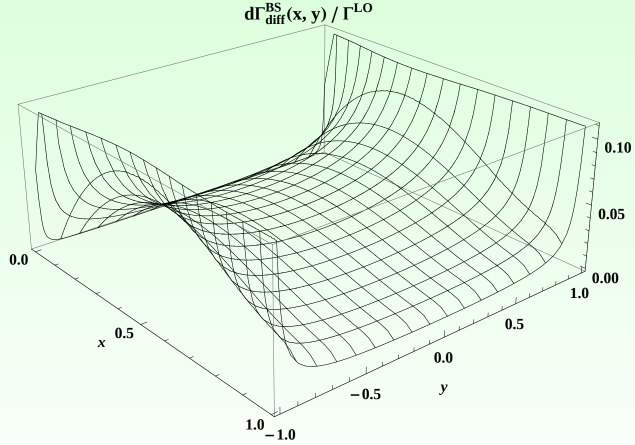
- thus we get the IR finite result for BS contribution

$$\Delta^{\mathrm{BS}}(x^{\mathrm{cut}}) = 2\int_{x^{\mathrm{cut}}}^{1}\int_{0}^{\sqrt{1-\nu^2/x}} \frac{\mathrm{d}\Gamma^{\mathrm{BS}}_{\mathrm{diff}}(x,y)}{\Gamma^{\mathrm{LO}}\left(\pi^0\to e^+e^-\right)}$$

Size of the correction for the KTeV cut

$$\Delta^{\mathrm{BS}}(0.95) = (0.30\pm0.01)\,\%$$

Differential decay width
Graph



New fit of the coupling $\chi^{(\mathrm{r})}$ value

- NLO radiative corrections in the QED sector did not solve the discrepancy using the contact interaction coupling finite part set to the value

$$\chi^{(\mathrm{r})}(M_\rho) = 2.2 \pm 0.9$$

$\rightarrow$ theoretically modeled by the LMD approximation to the large-N_C spectrum of vector meson resonances by *Knecht et al.*, 1999 [9]

- we can numerically fit the coupling to the result of the KTeV experiment using all available corrections
- alternatively, we can use the approximative formula to get the same result

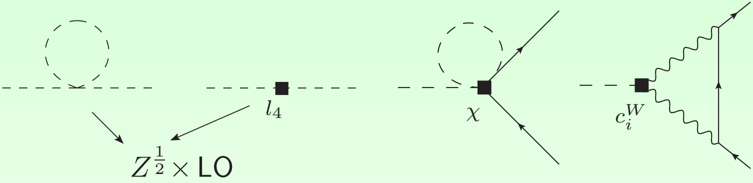
$$\chi^{(\mathrm{r})}(M_\rho) \overset{\text{eff}}{=} \frac{5}{2} + \frac{3}{2}\log\left(\frac{M_\rho^2}{m^2}\right) - \frac{\pi^2}{12} - \frac{1}{4}\log^2\left(\frac{M^2}{m^2}\right)$$

$$+ \sqrt{\frac{1}{2}\left(\frac{\pi M}{\alpha m}\right)^2 \frac{B(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95)}{B(\pi^0 \to \gamma\gamma)\left[1 + \delta^{(\text{2-loop})}(0.95)\right]} - \frac{\pi^2}{4}\log^2\left(\frac{M^2}{m^2}\right)}$$

Final model independent effective value (incl. higher orders, alt. a new physics)

$$\chi^{(\mathrm{r})}(M_\rho) = 4.5 \pm 1.0$$

One-loop diagrams of order α^2/F^3 for process $\pi^0 \to e^+e^-$

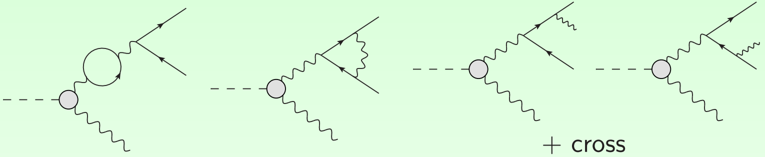


- the leading log estimation, i.e. taking terms $\sim \log^2 \mu^2$ (up to two loops) $\to$ Weinberg consistency relation
- only the contribution from c_{13}^W diagram survives

The final correction $\to$ stability in the strong sector

$$\Delta^{\mathrm{LL}}\chi^{(\mathrm{r})}(M_\rho) = \frac{1}{36}\left(\frac{M}{4\pi F}\right)^2 \left(1 - \frac{10m^2}{M^2}\right) \log^2\left(\frac{M_\rho^2}{m^2}\right) \doteq 0.081$$

Dalitz decay radiative corrections



(a) (b) (c) (d)

- investigation of the radiative correction to the decay $\pi^0 \to e^+e^-\gamma$
- motivated by needs of data analysis in NA48/NA62 experiments at CERN
- one of the goals is to measure the slope a of the $\pi^0\gamma\gamma^*$ transition form-factor
- corrections to the Dalitz plot published by *Mikaelian and Smith*, 1972 [10] in the form of a table of values → not dense and precise enough, programme lost
- the calculation was done again, without any approximation unlike before
  → even though it was possible to significantly simplify the result
  → can be used also for the related decay $\eta \to \mu^+\mu^-\gamma$ etc.
- a C/F77 code was written, which returns the correction for any given x and y
  → propagated into the standard simulation software of the NA62 experiment
- *to be published soon*

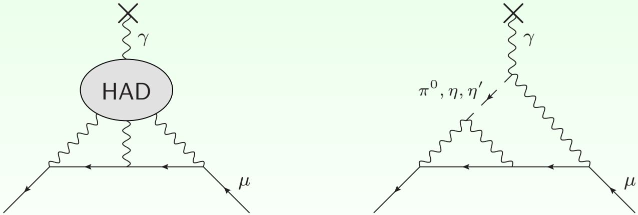
Outlook

Pseudoscalar decays

- $\chi^{(\mathrm{r})}$ universal for $P \to l^+l^-$ processes up to $\mathcal{O}(m_l^2/\Lambda_{\chi\mathrm{PT}}^2)$
- if the measurements are not compatible with the result $\chi^{(\mathrm{r})}(M_\rho) = 4.5 \pm 1.0$, something is not under control, maybe a new physics is present

Muon $g-2$: hadronic light-by-light scattering

- pseudoscalar meson exchange contribution to g-2 is highly model dependent
- interplay between precise measurements and theoretical calculations of presented decays represents a valuable feedback for modeling the $F_{\pi^0\gamma^*\gamma^{(*)}}$ form-factor



(a) HLbL scattering general contribution

(b) Pseudoscalar meson exchange (contains $\pi^0\gamma^*\gamma^{(*)}$ vertices)

Thank you for your attention!

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