

A physical description of coffee cooling in a pot

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One everyday phenomenon – coffee getting cold in a pot – is studied as an example of a typical method used in physics. We meet this topic in many articles in which authors usually limit themselves on using of Newtons law of cooling (see references^{1,2,3,4,5,6,7,8,9}). Our effort is aimed at the non-professional audience, we suppose only elementary knowledge of the subject and our approach should be obvious in the common sense.

The obvious beginning is to focus attention to the phenomenon and to collect some experience. The second step is the experiment in controlled conditions and with different variants of studied objects. The third step is an attempt to understand the results of the experiment and the nature of the phenomenon. To reach that in the case of cooling coffee, we try to follow the pathway of heat in the studied system within a sequence of models realized as spreadsheets in Excel.

We can guess that some heat is used for evaporation, some heat is radiated to surroundings and other amount of heat accepted from surroundings. We also must not forget the transport of heat between liquid and pot, through the pot and finally between pot and surroundings. We do not measure the individual heat flows, but we put them into the model and from the comparison of the model prediction for the time-dependent temperature with the data we deduce the role of different mechanisms and the parameters which enter the model.

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Introduction

A standard situation from the everyday life is a pot of coffee slowly getting cold when we forget about it (see fig. 1). Does it matter what kind of the pot we use? What are the physics processes which play some role in this ordinary occurrence? Which are more important, which less? Can we influence the evolution in the direction we need, like cool faster to drink in a hurry or cool slower during our busy activity? Let us turn to experiments. At first we measure the time-dependent temperature of the cooling coffee in a china pot on a table. The measurement should be done in reasonable intervals, e.g. in every five

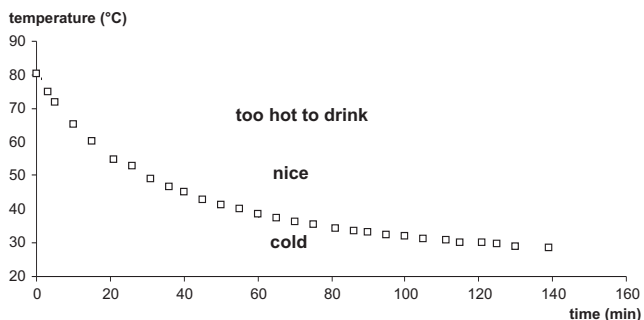


FIG. 1: The measured cooling of coffee. The personal views of the optimal drinking temperature could be different, of course.

minutes, either with a usual mercury thermometer (our case) or with some more sophisticated equipment. The estimated error of our measurement is about 0.3°C. We would like to concentrate to the detailed understanding of the measured dependence and to identifying the relevant physics phenomena contributing to the process. We will try to achieve this goal by constructing a suitable simulation model and by refining it to fit the data.

I. THE FIRST MODEL

We will start with a simple assumption that the heat transferred from the hot coffee to the colder surroundings per unit time is dependent only on the temperature difference and that this dependence is linear:

$$\Delta Q / \Delta \tau = k(t_c - t_s),$$

where k is the coefficient of the heat exchange between coffee and surroundings, t_c is the changing temperature of the coffee, t_s is the constant temperature of surroundings. We assume that the temperature of the coffee is uniform. Than we can write the equation relating the heat loss with the change of the coffee temperature with time τ in the form

$$m c \frac{\Delta t_c}{\Delta \tau} = \frac{\Delta Q}{\Delta \tau} = k(t_c - t_s)$$

where m is the mass of coffee and c is the specific heat capacity of it. This relates to Newtons law of cooling.

This equation we approximately and naively solve in the simplest possible manner doing small steps in time with the help of Excel (ignoring all the knowledge on solution of differential equations), we only check that the time step is sufficiently small not to influence the results. We fitted the coefficient k to meet the data (using Solver from Excel). As we may conclude from fig. 2, the first

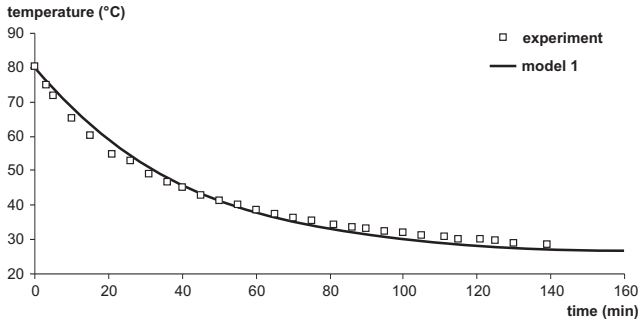


FIG. 2: The time-dependent temperature from the experiment confronted with the results of the first model (fitted $k = 0.54 \text{ J.s}^{-1}.\text{K}^{-1}$).

model systematically underestimates the heat loss at the early moments and overestimates it later.

II. THE SECOND MODEL

To improve the agreement of the model and data we will try to use the "brute force method" - we introduce the arbitrary exponent x into the heat transfer: $\Delta Q/\Delta\tau = k(t_c - t_s)^x$. The introduction of this exponent is clearly a substitute for better and more detailed description of the process. Both parameters k and x are fitted with Excel. The fig. 3 shows better accuracy of the second model. The quality of the fit we judge from the value of residual sum of squares divided by the number of degrees of freedom - " χ^2/NDF ", see any textbook on statistical methods, e.g.¹⁰. Values much larger than 1 signalize the poor fit.

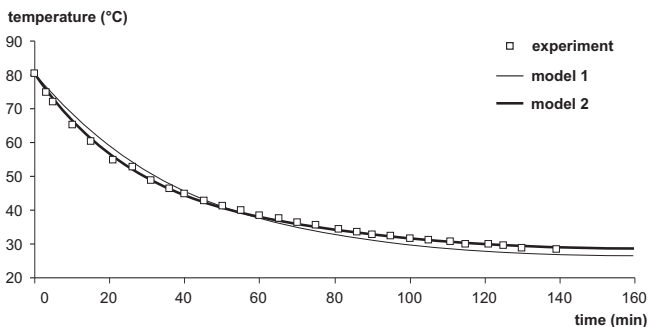


FIG. 3: The measured temperature and the both models. Fitted $k = 0.17 \text{ J.s}^{-1}.\text{K}^{-x}$, $x = 1,34$. χ^2/NDF changed from 32.1 for the first model to 2.4 for the second model.

What are the effects which are mimicked by the power

in the heat transfer? We may expect heat losses by evaporation and related loss of coffee mass. Heat could be radiated to surroundings and accepted from it. The pot is heated first and then it cools, it may have the temperature different from coffee. Heat is transferred through all the surfaces. Maybe there are other effects still missing in our list

III. THE THIRD MODEL

We will try to introduce all the abovementioned effects:

- transfer of heat from coffee to the pot:
 $\Delta Q_1/\Delta\tau = k_1(t_c - t_p)$;
- coffee evaporation to surroundings:
 $\Delta Q_2/\Delta\tau = vl_v t_c P$,
where v is the speed of evaporation, l_v is specific heat of vaporization, P open surface;
- radiation from coffee to surroundings:
 $\Delta Q_3/\Delta\tau = \alpha\sigma T_c^4 P$,
where σ is the Stefan-Boltzmann constant, α is the absorptivity of coffee;
- radiation from surroundings to coffee:
 $\Delta Q_4/\Delta\tau = \alpha\sigma T_s^4 P$
- transfer of heat from pot to surroundings:
 $\Delta Q_5/\Delta\tau = k_2(t_p - t_s)$;
- radiation from pot to surroundings:
 $\Delta Q_6/\Delta\tau = \beta\sigma T_p^4 S$,
where S is the emitting surface of the cup, β is the absorptivity of its material;
- radiation from surroundings to pot:
 $\Delta Q_7/\Delta\tau = \beta\sigma T_s^4 S$;

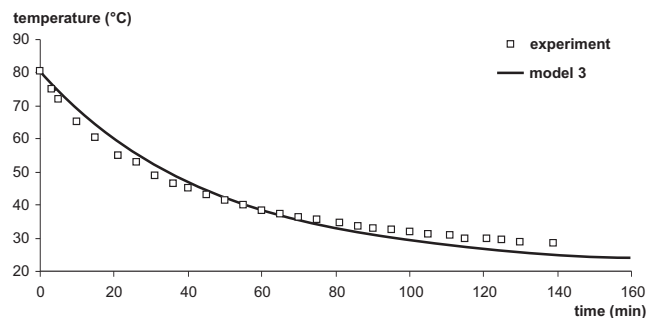


FIG. 4: The measured temperature and the third model. The fitted constants are $k_1 = 31.43 \text{ J.s}^{-1}.\text{K}^{-1}$, $k_2 = 0.31 \text{ J.s}^{-1}.\text{K}^{-1}$

Our model 3 (see fig. 4) is not yet ideal - the pot is still thin (i. e. it has only one temperature). But that model is more "physical" than the former - at least coffee evaporates. We are lacking any insight into the details of evaporation, so we employed the linear dependence on

t_c . All the parameters α , β , v , k_1 , k_2 are fitted within reasonable bounds. We are not much successful with this model - the residual sum $2/\text{NDF} = 81.3$ compared to $2/\text{NDF} = 32.1$ for the first model. Before turning to the next model we include one more experimental datum - the change of coffee weight = the weight of evaporated liquid. With this datum included we have $\chi^2/\text{NDF} = 81.6$.

The lack of faster cooling at the beginning persists in the model. Maybe the evaporation speed linear in temperature is unrealistic?

IV. THE FOURTH MODEL

Although we do not have any detailed knowledge on the dependence of evaporation speed on temperature and other conditions, we can try to put a free power z in the "vaporising" term:

$$\Delta Q_2/\Delta\tau = vl_v t_c^z P.$$

The fit results in $z = 3.73$! Then the agreement of the model and data is remarkable (see fig. 5) - $\chi^2/\text{NDF} = 1.0$.

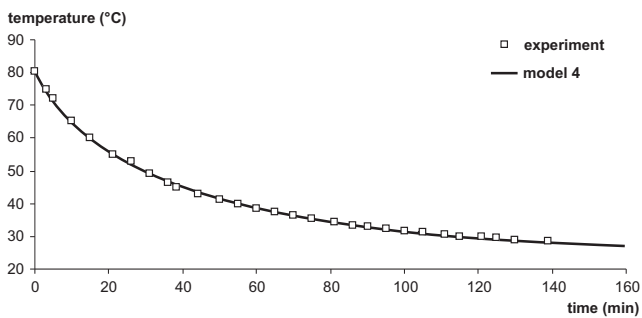


FIG. 5: The measured temperature and the fourth model. The fitted constants are $k_1 = 29.87 \text{ J.s}^{-1}.\text{K}^{-1}$, $k_2 = 0.18 \text{ J.s}^{-1}.\text{K}^{-1}$

But the natural question appears: Is the funny exponent 3.73 reasonable? We have made with our colleagues a simple dedicated experiment for measuring of the evaporation speed, see fig. 6. The result of the experiment, power $z = 3.7$, really agrees with the above value!

V. THE FIFTH MODEL

In the next model we suppose the pot with nonzero thickness to treat different pots. It means we assume there are an external and an internal temperature of the pot. Then we need to adapt the two already used terms in the following way:

$$\Delta Q_1/\Delta\tau = k_1(t_c - t_{pi}),$$

$$\Delta Q_5/\Delta\tau = k_2(t_{pe} - t_s),$$



FIG. 6: The measurement of the evaporation speed: The immersion heater controlled by the simple switching circuit act as thermostat, the pot with water is regularly weighted.

where t_{pi} , t_{pe} are the internal and the external temperature of the pot. We must also introduce another term in our model - heat transfer through the pot:

$$\Delta Q_8/\Delta\tau = k_3(t_{pi} - t_{pe})$$



FIG. 7: The three various experimental pots.

VI. THE COMPARISON OF THE INFLUENCE OF DIFFERENT PARAMETERS

By introduction of the parameters in the sequence of our models and by fitting them to meet best the data we get sets of parameters, which describe the experiment

comparatively well. Clearly we need further experiments with exactly directed setup for specification of different parameters. To check the importance of the evaporation we put some fat on the liquid surface. We use various pots with various specific heat capacities and the heat conductivities, see fig. 7. Comparison of dark coffee and clear water shows us the role of the absorptivity of the liquid surface. We performed a set of experiments with different conditions: At first we discovered the results do not depend on the surface tension what is important for the comparison of coffee and water. The next step was measurement for:

- three pots (china, ordinary metal sheet pot and vacuum-pot, it means the pot with double wall)
- three liquids (coffee, water and water with fat on its surface)

TABLE I: The fitted parameters of the fifth model.

Liquid	Pot	α	β	$v \cdot 10^{11}$ [$\frac{\text{kg} \cdot \text{°C}}{\text{s} \cdot \text{m}^2}$]	k_1 [$\frac{\text{J}}{\text{s} \cdot \text{K}}$]	k_2 [$\frac{\text{J}}{\text{s} \cdot \text{K}}$]	k_3 [$\frac{\text{J}}{\text{s} \cdot \text{K}}$]
coffee	china	0.990	0.192	15.8	2.90	0.360	2.04
water	china	0.005	0.192	15.8	2.90	0.360	2.04
w.w.fat	china	0.005	0.192	1.94	2.90	0.360	2.04
coffee	metal	0.990	0.178	15.8	2.90	0.360	2.42
water	metal	0.005	0.178	15.8	2.90	0.360	2.42
w.w.fat	metal	0.005	0.178	1.94	2.90	0.360	2.42
coffee	vac.pot	0.990	0.050	15.8	2.90	0.360	0.187
water	vac.pot	0.005	0.050	15.8	2.90	0.360	0.187
w.w.fat	vac. pot	0.005	0.050	1.94	2.90	0.360	0.187

By determination and matching of all coefficients we have justified following facts (look at values at table I):

- coffee surface (dark) radiates and absorbs heat better than water surface
- shining stainless pot radiates and absorbs heat worse than the dim one
- fat on the liquid surface prevents evaporation
- heat transfers through the china pot a little worse than through the thin metal pot and much better than through the vacuum-pot

VII. CONCLUSION

This contribution shows the beginning of a story originating from one phenomenon of our everyday life. We formulated few models to describe the cooling down of a cup of coffee with rather good final result. The sequence of approximations shows how we are pushed towards the more detailed description and towards the study which processes are important and which not. We were forced to introduce some nonlinearity; we tried to do it by introducing power into evaporation speed. We verified its power dependence on the temperature in an independent experiment. The whole story shows by simple means the path from observation to the description on some level and may continue in further studies.

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