Baryon Resonances in Effective Field Theory models

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Outline

• Introduction

• EFT calculation of $\pi+N \rightarrow N+e^+e^-$

• Problems related to higher spins

• Electromagnetic form factors – VMD
Introduction

Theory of strong interaction = QCD
- a theory of quarks and gluons
- in nature: hadrons, quark confinement
- we don’t know how to derive hadron physics from QCD
  → need for Effective Field Theories (EFT)

Effective Field Theories:
- fields for various hadron types
- symmetries of strong interaction are respected
- non-renormalizable → usually only tree level calculations
- contains no info on the internal structure of hadron
  → phenomenological form-factors, width parametrizations
    to account for internal structure and higher order effects
- unitarity is violated: cross sections increase at high energies
EFT calculation of $\pi+N \rightarrow N+e^+e^-$

- Form factors for Born terms
- Phenomenological width parametrizations
- Special cutoff for u-channel resonance diagrams
- 16 baryon resonances up to $\sim 2$ GeV and spin-5/2
Parameters of the model

- From PDG
- Fit to pion photoproduction (RN$_\gamma$ coupling constants)

![Graphs showing destructive interference](image-url)
Results for $\pi+N \rightarrow N+e^+e^-$ – dilepton spectra

RN$\gamma$ coupling: VMD with only $\rho^0$
Results for $\pi+N \rightarrow N+e^+e^-$ – dominant channels

$\pi^- + p, \quad \sqrt{s} = 1.9 \text{ GeV}$
Problems with higher spin

Higher spin baryons are usually described by Rarita-Schwinger spinors

These contain also lower spin degrees of freedom $\rightarrow$ constraints (= Rarita-Schwinger relations):

- totally antisymmetric
  \[
  \psi_{\mu_1 \cdots \mu_n} = \psi_{\mu_n \cdots \mu_1} 
  \]
- traceless
  \[
  \psi_{\alpha \mu_3 \cdots \mu_n} = 0 
  \]

Projector to spin-3/2 degrees of freedom:

\[
P_{3/2}^{\mu\nu} (p, m_R) = -(\not{p} + m_R) \left( g^{\mu\nu} - \frac{\gamma^\mu \gamma^\nu}{3} - 2 \frac{p^\mu p^\nu}{3 m_R^2} + \frac{p^\mu \gamma^\nu - p^\nu \gamma^\mu}{3 m_R} \right)
\]

- continuation outside the mass-shell is problematic

Off-shell parameters, e.g. $\Delta N\pi$ interaction:

\[
\mathcal{L}_{\text{int}} = g \bar{\psi}_\mu (g^{\mu\nu} + a \gamma^\mu \gamma^\nu) \Psi \partial_\nu \phi + \text{H.c.}
\]
• **Velo-Zwanziger (VZ) problem:**
  faster than light propagation of interacting spin $\geq 1$ field

• **Johnson-Sudarshan (JS) problem:**
  commutation relations and relativistic covariance are not compatible for spin-$3/2$

• The VZ and JS problems are connected to the presence of
  lower spin degrees of freedom

• Interactions can change the constraints $\rightarrow$ construction of interacting theories with
  spin-$>1/2$ fermions is non-trivial
Pascalutsa: Dirac-Faddeev quantization of interacting spin-3/2 particles

- Dirac: Generalized Hamiltonian Dynamics
  - Hamilton formalism for constrained systems → D.O.F. counting
- The naïve interaction Lagrangians for higher-spin baryons are inconsistent

Consistent interactions are invariant under the gauge transformation:

\[ \psi_\mu \rightarrow \psi_\mu + \partial_\mu \epsilon \]

\( \epsilon \): spin-1/2 field

Such interactions can be constructed in terms of the R.S. field strength tensor, and its dual:

\[ G^{\mu \nu} = \partial^\mu \psi^\nu - \partial^\nu \psi^\mu \]

\[ \tilde{G}^{\mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} G_{\rho \sigma} \]
Higher spin – traditional vs. gauge-invariant interactions

e.g. pion photoproduction:

Traditional $\Delta N\pi$ interaction:

$$\mathcal{L}_{R_{3/2}N\pi} = \frac{g_{RN\pi}}{m_{\pi}} \bar{\psi}_R^{\mu} \Gamma \bar{\tau} \psi_N \cdot \partial_{\mu} \bar{\pi} + \text{H.c.}$$

A cutoff is needed for u-channel diagrams of the form:

$$F_u(s) = \left( \frac{\Lambda_u^2}{\Lambda_u^2 + q^2} \right)^J$$

$$q^2 = \frac{\lambda(s, m_N^2, m_{\pi}^2)}{4s}$$

a function of $\sqrt{s}$!

Resonance: mixture of pure 3 quark state and $\pi N$ state

Gauge-invariant $\Delta N\pi$ interaction:

$$\mathcal{L}_{\text{int}}^{(\text{GI})} = g \varepsilon^{\mu\nu\alpha\beta} \left( \partial_{\mu} \bar{\psi}_v \right) \gamma_5 \gamma_\alpha \Psi \partial_\beta \phi + \text{H.c.}$$

Fernández-Ramírez et al.: the above cutoff is not needed if GI interaction is used but: they also implement final state interactions to restore unitarity
Higher spin – traditional vs. gauge-invariant interactions

Pion photoproduction:

Born contribution (with FFs) + Δ contributions (s- and u-channel; no cutoff for u-channel)

Gauge invariant interactions:

- behave better at relatively low energy
- diverge faster at high energy (more derivatives – higher power of momenta)
- unitarization is still needed
Two possibilities for the $V\gamma$ coupling:

\[ \mathcal{L}_{V\gamma} = -\frac{e}{2g_V} F_{\mu\nu}^\gamma V_{\mu\nu} \quad F_{VMD}(k^2) = -\frac{e}{g_V k^2 - m_V^2 + i\sqrt{k^2} \Gamma_V(k^2)} \]

- RNV coupling describes both $R \rightarrow NV$ and $R \rightarrow N\gamma$ decays
- Incompatible: the NV channel usually needs a stronger RNV coupling

\[ \tilde{\mathcal{L}}_{V\gamma} = -\frac{em_V^2}{g_V} A^\mu V_{\mu} \]

- Gauge invariant
- Direct photon coupling is needed for $R \rightarrow N\gamma$ decay
Vector Meson Dominance – E.M. form factors

ω is subthreshold for most resonances → RNω coupling is hard to get

Interference of ρ and ω contribution is important

Isospin structure of RNV Lagrangians:

$$\mathcal{L}_{N^* N\rho} \sim g \bar{N^*} \vec{\tau} N \cdot \vec{\rho} + \text{H.c.}$$

$$= g \bar{N^*}^+ p \rho^0 + \sqrt{2} g \bar{N^*}^+ n \rho^+ + \sqrt{2} g \bar{N^*}^0 p \rho^- - g \bar{N^*}^0 n \rho^0 + \text{H.c.}$$

$$\mathcal{L}_{N^* N\omega} \sim g' \bar{N^*} \vec{\tau} N \cdot \vec{\rho} + \text{H.c.} = g' \bar{N^*}^+ p \omega + g' \bar{N^*}^0 n \omega + \text{H.c.}$$

→ both constructive and destructive interference occurs

Extended Vector Meson Dominance (Krivoruchenko and Martemyanov):
include excited states of vector mesons ρ', ρ''..., and get the correct asymptotic behavior of baryon form factors according to quark counting rules
Thank you!
Differential cross section:

\[
\frac{d\sigma}{dM} = \frac{M}{64(2\pi)^4 s} \frac{|k|}{|q|} \int d(cos \theta_k) d\Omega_{k_1} \frac{1}{n_{pol}} \sum_{pol} |M|^2
\]

The electromagnetic part of the matrix element can be calculated separately:

\[
M = -\frac{e}{k^2} M^\text{had}_\mu \bar{u}(k_1) \gamma^\mu v(k_2)
\]

\[
\sum_{pol} |M|^2 = \frac{e^2}{k^4} W_{\mu\nu} l^{\mu\nu}
\]

\[
l^{\mu\nu} = 4 \left( k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu} \right)
\]

\[
W_{\mu\nu} = \sum_{pol} M^\text{had}_\mu M^\text{had}_\nu^* \\
\text{hadronic tensor}
\]

\[
\text{leptonic tensor}
\]
Gauge invariance: \[ \mathcal{M}^\text{had}_{\mu} k^\mu = 0 \]

This will be satisfied only by the sum of the Born contributions, not by the individual diagrams.

Interactions with photon and \( \rho \)-meson from gauge covariant derivatives:
(because VMD is used)

\[ \nabla_\mu = \partial_\mu + ieA_\mu Q_i - i\tilde{g}_\rho \rho_\mu \cdot \tilde{T} \]

\[ \mathcal{L}_{NN\gamma} = -e\bar{\psi}_N \left[ \frac{1 + \tau_3}{2} A - \left( \kappa^s + \kappa^v \tau_3 \right) \frac{\sigma_{\mu\nu}}{4m_N} F^{\mu\nu} \right] \psi_N \quad \text{from nucleon kinetic term} \]

\[ \mathcal{L}_{\gamma\pi\pi} = -eA_\mu J^\mu_\pi \quad J^\mu_\pi = i(\pi^- \partial^\mu \pi^+ - \pi^+ \partial^\mu \pi^-) \quad \text{from pion kinetic term} \]

\[ \mathcal{L}_{NN\pi\gamma} = -\frac{ief_{NN\pi}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma^\mu \gamma^\nu \psi_N \cdot A_\mu Q^\nu \quad \text{contact interaction} \]

\[ \mathcal{L}_{NN\pi} = -\frac{f_{NN\pi}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma^\mu \gamma^\nu \psi_N \cdot A_\mu \pi^\nu \]

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Couplings to the $\rho$-meson:

\[
\mathcal{L}_{NN\rho} = \frac{\tilde{g}_\rho}{2} \bar{\psi}_N \left( \vec{\phi} - \kappa_\rho \frac{\sigma_{\mu\nu}}{4m_N} \vec{\rho}_{\mu\nu} \right) \cdot \vec{\tau} \psi_N
\]

\[
\mathcal{L}_{\rho\pi\pi} = -\tilde{g}_\rho \left[ (\partial_\mu \vec{\pi}) \times \vec{\pi} \right] \cdot \vec{\rho}_\mu
\]

\[
\mathcal{L}_{NN\pi\rho} = -\frac{\tilde{g}_\rho f_{NN\pi}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma^\mu \vec{\tau} \psi \cdot (\vec{\rho}_\mu \times \vec{\pi})
\]

from nucleon kinetic term + tensor coupling

from pion kinetic term

contact interaction from the PV pion-nucleon coupling

coupling constants to nucleon and pion are related:

\[
2g_{NN\rho} = g_{\rho\pi\pi} = \tilde{g}_\rho
\]

eperimental values:

\[
g_{\rho\pi\pi} = 5.96
\]

\[
g_{NN\rho} = 2.6
\]

\[
g_{\rho\pi\pi}/g_{NN\rho} = 2.29
\]
• Only the sum of the Born diagrams is gauge invariant
  → gauge invariance is lost if different form factors are used for each diagram

• But: it can be proved, that
  non-gauge invariant terms are free from poles
  → they can be generated by suitable contact terms in the Lagrangian
  (for pion photoproduction: Davidson & Workman, 2001)

\[ \Delta T^\text{Born}_\mu = \frac{\sqrt{2}e f_{NN\pi}}{m_\pi} 2m_N \gamma_5 \left[ \left( \hat{F}(s, u, t) - F_3(t) \right) \frac{2q^\mu - k^\mu}{t - m_\pi^2} - \left( \hat{F}(s, u, t) - F_2(u) \right) \frac{2p_i^\mu - k^\mu}{u - m_N^2} \right] \]

\[ \hat{F}(s, u, t) = F_1(s) + F_2(u) + F_3(t) - F_1(s)F_2(u) - F_1(s)F_3(t) - F_2(u)F_3(t) + F_1(s)F_2(u)F_3(t) \]

• poles are removed from \( \Delta T^\text{Born}_\mu \)
• \( T^\text{Born}_\mu \) is gauge invariant

Form factors:

\[ F_1(s) = \frac{1}{1 + (s - m_N^2)^2/\Lambda^4} \]
\[ F_2(u) = \frac{1}{1 + (u - m_N^2)^2/\Lambda^4} \]
\[ F_3(t) = \frac{1}{1 + (t - m_\pi^2)^2/\Lambda^4} \]
Manifestly gauge invariant form of $T^\text{Born}_\mu$:

$$T_\mu = \sum_{i=1}^{4} A_i M_{i,\mu}$$

$$M_{1,\mu} = \gamma_5 (\gamma_\mu k - k_\mu),$$

$$M_{2,\mu} = \frac{\gamma_5}{2} \left[ (2p_{i\mu} - k_\mu)(2q \cdot k - M^2) - (2q_\mu - k_\mu)(2p_i \cdot k - M^2) \right],$$

$$M_{3,\mu} = \frac{\gamma_5}{2} \left[ \gamma_\mu (2p_f \cdot k + M^2) - (2p_{f\mu} + k_\mu) k \right],$$

$$M_{4,\mu} = \frac{\gamma_5}{2} \left[ \gamma_\mu (2p_i \cdot k - M^2) - (2p_{i\mu} - k_\mu) k \right].$$

$$A_1 = -\frac{\sqrt{2}e f_{NN\pi}}{m_\pi} \left[ \frac{1}{2m_N} (F_1 \kappa_n + F_2 \kappa_p) + \frac{2m_N F_2}{u - m_N^2} (1 + \kappa_p) + \frac{2m_N F_1}{s + m_N^2} \kappa_n \right],$$

$$A_2 = \frac{\sqrt{2}e f_{NN\pi}}{m_\pi} \frac{4m_N \hat{F}}{(t - m_\pi^2)(u - m_N^2)},$$

$$A_3 = \frac{\sqrt{2}e f_{NN\pi}}{m_\pi} \frac{2\kappa_n F_1}{s - m_N^2},$$

$$A_4 = \frac{\sqrt{2}e f_{NN\pi}}{m_\pi} \frac{2\kappa_p F_2}{u - m_N^2}.$$
The whole procedure has to be repeated for the VMD contributions!

Split off the VMD form factor:

\[ M_{\mu, \text{had, VMD}} = F_{\text{VMD}}(k^2) \tilde{M}_{\mu} \]

\[ F_{\text{VMD}}(k^2) = - \frac{e}{g_\rho} \frac{k^2}{k^2 - m_\rho^2 + i\sqrt{k^2} \Gamma_\rho(k^2)} \]

Split off the nucleon spinors:

\[ \tilde{M}_{\mu} = \bar{u}_f \gamma_\mu u_i \]

\[ \Delta \tilde{T}_{\mu}^{\text{Born, VMD}} = \frac{g_\rho f_{NN} \pi}{\sqrt{2m_\pi}} 2m_N \gamma_5 \]

\[ \times \left[ \left( \hat{F}(s, u, t) - F_2(u) \right) \frac{2p_i^\mu - k_\mu}{u - m_N^2} - \left( \hat{F}(s, u, t) - F_1(u) \right) \frac{2p_f^\mu + k_\mu}{s - m_N^2} - 2 \left( \hat{F}(s, u, t) - F_3(t) \right) \frac{2q^\mu - k_\mu}{t - m_\pi^2} \right] \]

\[ \Delta \tilde{T}_{\mu}^{\text{Born, VMD}} \] has no poles, \[ \tilde{T}_{\mu}^{\text{Born, VMD}} \] is gauge invariant
Gauge invariant form of the matrix element:

\[ T_{\mu}^{\text{Born, VMD}} = \sum_{i=1}^{5} \hat{A}_i M_{i,\mu} \]

One more gauge invariant combination:

\[
M_{5,\mu} = \frac{\gamma_5}{2} \left[ (2p_f \gamma_{\mu} + k_{\mu})(2q \cdot k - M^2) - (2q_{\mu} - k_{\mu})(2p_f \cdot k + M^2) \right]
\]

\[
\hat{A}_1 = \frac{\tilde{g}_\rho f_{NN\pi}}{\sqrt{2}m_\pi} \left[ \frac{\kappa_\rho}{2m_N} (F_2 - F_1) + 2m_N (1 + \kappa_\rho) \left( \frac{F_2}{u - m_N^2} - \frac{F_1}{s - m_N^2} \right) \right]
\]

\[
\hat{A}_2 = -\frac{\tilde{g}_\rho f_{NN\pi}}{\sqrt{2}m_\pi} \frac{4m_N \hat{F}}{(t - m_\pi^2)(u - m_N^2)},
\]

\[
\hat{A}_3 = \frac{\tilde{g}_\rho f_{NN\pi}}{\sqrt{2}m_\pi} \frac{2\kappa_\rho F_1}{s - m_N^2},
\]

\[
\hat{A}_4 = -\frac{\tilde{g}_\rho f_{NN\pi}}{\sqrt{2}m_\pi} \frac{2\kappa_\rho F_2}{u - m_N^2},
\]

\[
\hat{A}_5 = -\frac{\tilde{g}_\rho f_{NN\pi}}{\sqrt{2}m_\pi} \frac{4m_N \hat{F}}{(t - m_\pi^2)(s - m_N^2)}.
\]
Resonances – Lagrangians

$R\pi\pi$ interactions for various spin-parity combinations:

\[
\mathcal{L}_{R_{1/2}N\pi} = -\frac{g_{R\pi\pi}}{m_\pi} \bar{\psi}_R \Gamma \gamma^\mu \tau \psi_N \cdot \partial_\mu \bar{\pi} + \text{h.c.}
\]

\[
\mathcal{L}_{R_{3/2}N\pi} = \frac{g_{R\pi\pi}}{m_\pi} \bar{\psi}_R \Gamma \psi_N \cdot \partial_\mu \bar{\pi} + \text{h.c.}
\]

\[
\mathcal{L}_{R_{5/2}N\pi} = \frac{g_{R\pi\pi}}{m_\pi} \bar{\psi}_R^{\mu\nu} \Gamma \psi_N \cdot \partial_\mu \partial_\nu \bar{\pi} + \text{h.c.}
\]

\[\Gamma = \gamma_5 \quad \text{or} \quad \Gamma = 1\]

depending on spin and parity
Resonances – Lagrangians

$\mathcal{L}_{R_{1/2}N_\gamma} = \frac{g_{RN\gamma}}{2m_\rho} \bar{\psi}_R \sigma^{\mu\nu} \tilde{\Gamma} \psi_N F_{\mu\nu} + \text{h.c.},$

$\mathcal{L}_{R_{1/2}N_\rho} = \frac{g_{RN\rho}}{2m_\rho} \bar{\psi}_R \bar{\sigma}^{\mu\nu} \tilde{\Gamma} \psi_N \cdot \bar{\rho}_{\mu\nu} + \text{h.c.},$

$\mathcal{L}_{R_{3/2}N_\gamma} = -\frac{ig_{RN\gamma}}{m_\rho} \bar{\psi}_R \gamma^\mu \tilde{\Gamma} \psi_N F_{\mu\nu} + \text{h.c.},$

$\mathcal{L}_{R_{3/2}N_\rho} = -\frac{ig_{RN\rho}}{m_\rho} \bar{\psi}_R \bar{\gamma}^\mu \tilde{\Gamma} \psi_N \cdot \bar{\rho}_{\mu\nu} + \text{h.c.},$

$\mathcal{L}_{R_{5/2}N_\gamma} = -\frac{ig_{RN\gamma}}{m_\rho} \bar{\psi}_R^{\mu\rho} \gamma^\nu \tilde{\Gamma} (\partial_{\rho} \psi_N) F_{\mu\nu} + \text{h.c.},$

$\mathcal{L}_{R_{5/2}N_\rho} = -\frac{ig_{RN\rho}}{m_\rho} \bar{\psi}_R^{\mu\rho} \bar{\gamma}^\nu \tilde{\Gamma} (\partial_{\rho} \psi_N) \cdot \bar{\rho}_{\mu\nu} + \text{h.c.}$
Width parametrizations – form factors for resonances

Width of $R \rightarrow N\pi$:

$$\Gamma(p^2) = \Gamma(m_R^2) \frac{m_R}{\sqrt{p^2}} \left( \frac{q}{q_R} \right)^{2l+1} \left( \frac{q_R^2 + \delta^2}{q^2 + \delta^2} \right)^{l+1}$$

$q$ ($q_R$): outgoing three-momentum (at the resonance pole)

$l$: angular momentum of the pion

cutoff parameter:

$$\delta^2 = (m_R - m_N - m_{\pi(\eta)})^2 - \frac{[\Gamma(m_R^2)]^2}{4}$$

Form factor at $RN\pi$ vertices:

$$F(p^2) = \sqrt{\frac{m_R}{\sqrt{p^2}}} \left( \frac{q_R^2 + \delta^2}{q^2 + \delta^2} \right)^{\frac{l+1}{2}}$$

→ Feynman diagram calculation gives a $\Gamma(p^2)$ similar to the above.
Width parametrizations – form factors for resonances

Two-pion decays of baryon resonances:

\[ R \to (\Delta/N(1440))\pi \to N\pi\pi \]
\[ R \to N(\rho/\sigma) \to N\pi\pi. \]

Width parametrization from Feynman diagram calculation

Cutoff factor:

\[ F_{\pi\pi}(p^2) = \left[ \frac{(\sqrt{p^2} - m_N - 2m_\pi)^2 + \delta^2}{(m_R - m_N - 2m_\pi)^2 + \delta^2} \right]^2 \]

Extra cutoff for \(u\)-channel resonance diagrams:

\[ F_u(p^2) = \left( \frac{\Lambda_u^2}{\Lambda_u^2 + q^2} \right)^J \quad \Lambda_u = 0.3 \text{ GeV} \]
Coupling constants of Baryon resonances

\( R N\pi \) and \( R N\rho \) coupling:
- from the partial decay widths
- data are from PDG (full width, branching ratio)

\( R N\gamma \) coupling:
- the \( N\gamma \) branching ratio is poorly known in many cases
- sign of coupling constants:
  - no info from decay widths
  - determine the sign of interference terms

\( \rightarrow R N\gamma \) coupling constants are determined from a fit of the total cross section of pion photoproduction
- \( R N\pi \) coupling constants are assumed to be positive
- sign of \( R N\gamma \) coupling constant is varied in the fit
# Baryon resonance parameters

<table>
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<th>$J^P$</th>
<th>$m_R$ (GeV)</th>
<th>$\Gamma_{\text{tot}}$ (MeV)</th>
<th>BR (%)</th>
<th>Coupling constants</th>
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