The density of available states of the DDHMS pre-equilibrium model

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Pre-equilibrium models

- Nucleons occupy single-particle states;

- The Fermi energy is between the last occupied and first unoccupied level of the ground state;

- Particles are nucleons above the Fermi energy and holes are unoccupied states below it. Both are called excitons.

The set of configurations with the same number of excitons is called an exciton class.
A reaction proceeds through energy-conserving two-nucleon collisions and eventual emission.

Configuration with $n$ excitons

Particle emission

Transition to configuration with $n$ excitons

Transition to configuration with $n \pm 2$ excitons
The exciton model

• The standard exciton model freezes the hole at the Fermi energy but assumes that the configurations with the same number of excitons are in equilibrium;

• It makes use of densities of states and transition densities, which greatly simplify the calculation.

• However, the assumption of equilibrium is questionable.

Exciton model transition rates for nucleon + $^{40}$Ca at incident energies of 25, 50 and 100 MeV.
The HMS model

• The HMS model ignores transitions between configurations with the same number of excitons;

• It treats transitions and emission from each particle and hole in a configuration individually – no equilibration – only $1p \rightarrow 2p1h$ and $1h \rightarrow 1p2h$ transitions;

• For a nucleon + $^{40}$Ca at 100 MeV, $10^9$ configurations versus 20 exciton classes – Monte Carlo simulation.

No equilibration?

- “Natural” model – transitions within a class and between classes have same magnitude;
- The no-mixing and “natural” model results are also almost identical;
- The complete-mixing and exciton model results are almost identical;
- Complete-mixing and no-mixing results differ ⇒ the exciton model requires complete mixing.

The DDHMS model

An analysis of linear-momentum $n$-exciton particle-hole states shows that the angular distribution of one of the particles or holes can be approximated by

$$p(\cos \theta) = \frac{2}{e^{a_n} - e^{-a_n}} e^{a_n \cos \theta}$$

Where the coefficient $a_n$ is proportional to the energy of the particle and inversely proportional to the exciton number $n$.


This expression is used to sample the scattering angle of each of the particles produced in a $1p \rightarrow 2p1h$ or $1h \rightarrow 1p2h$ transition after the energy has been determined by MC sampling.

The density of available states – 1p→2p1h

\[
\rho_{1p\rightarrow2p1h}(\vec{p}_1) = \frac{aV^2}{(2\pi\hbar)^6} \int \delta (\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta \left( \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{p_3^2}{2m} - \frac{p_4^2}{2m} \right) \\
\times \theta (p_{F2} - |\vec{p}_2|) d^3p_2 \theta (|\vec{p}_3| - p_{F1}) d^3p_3 \theta (|\vec{p}_4| - p_{F2}) d^3p_4 .
\]

This is the sum over all states for which:
1) the initial particle 1 collides with a particle 2 below the Fermi energy to furnish particles 3 and 4 above the Fermi energy;
2) in an energy and linear-momentum conserving reaction.

The form of the momenta of particles 1 and 2 (up to a rotation) is

\[
\begin{align*}
\vec{p}_{10} & = \frac{p_1}{|\vec{p}_1 + \vec{p}_2|} \begin{pmatrix}
-p_2 \sin \theta_{12} \\
0 \\
p_1 + p_2 \cos \theta_{12}
\end{pmatrix} \\
\vec{p}_{20} & = \frac{p_2}{|\vec{p}_1 + \vec{p}_2|} \begin{pmatrix}
p_1 \sin \theta_{12} \\
0 \\
p_2 + p_1 \cos \theta_{12}
\end{pmatrix}
\end{align*}
\]

when \( \vec{p}_{10} + \vec{p}_{20} = \begin{pmatrix} 0 \\
0 \\
|\vec{p}_1 + \vec{p}_2| \end{pmatrix} \).
The density of available states – $1p\rightarrow 2p1h$

We parametrize the final momenta in the same reference frame, again up to a rotation, as

$$\tilde{p}_{30} = \frac{p_3}{|\tilde{p}_3 + \tilde{p}_4|} U(\phi_{34}) \begin{pmatrix} -p_4 \sin \theta_{34} \\ 0 \\ p_3 + p_4 \cos \theta_{34} \end{pmatrix}$$

and

$$\tilde{p}_{40} = \frac{p_4}{|\tilde{p}_3 + \tilde{p}_4|} U(\phi_{34}) \begin{pmatrix} p_3 \sin \theta_{34} \\ 0 \\ p_4 + p_3 \cos \theta_{34} \end{pmatrix},$$

The volume element is unchanged but the integrals can be performed. We find, assuming equal Fermi momenta,

$$\rho_{1p\rightarrow 2p1h}(\tilde{p}_1) = \frac{aV^2}{(2\pi)^4} \frac{m}{\hbar^6 p_1} \left\{ (p_1^2 - 7p_F^2/5) p_F^3/3 + 2p_{2_{\text{min}}}^5/15 \right\}$$

where

$$p_{2_{\text{min}}}^2 = \max (0, 2p_F^2 - p_1^2)$$
Monte Carlo Calculations

Monte Carlo sampling of the momenta can be performed efficiently. We assume the Fermi energies to be equal and show calculations in which they are 35 MeV.

The Pauli blocking factor $\eta$

$$\eta = \frac{\rho}{\rho_{unblocked}}$$

Kikuchi and Kawai obtained (Nuclear matter and nuclear reactions)

$$\eta_{K-K} = \begin{cases} 
1 - \frac{7}{5}r & r < \frac{1}{2} \\
1 - \frac{7}{5}r + \frac{2}{5}(2 - 1/r)^{5/2} & r > \frac{1}{2}
\end{cases}$$

with

$$r = \frac{V}{p^2/2m}$$

We find

$$\eta = \eta_{K-K}/\left(1 + \frac{3}{5}r\right)$$

with $V$ being the Fermi energy.
Average particle and hole energy

We define

\[ e_h = e_F - \frac{p_h^2}{2m} \quad \text{and} \quad e_p = \frac{p_p^2}{2m} - e_F \]

From energy conservation,

\[ e_i = e_h + 2e_p \]

so that

\[ \langle e_p \rangle = \frac{e_i - \langle e_h \rangle}{2} \]

The average energy of occupied states below the Fermi energy is

\[ \langle e_{\text{occ}<e_F} \rangle = \frac{3}{5}e_F \]

As the incident energy increases, the occupied states are sampled more uniformly, so that

\[ \langle e_h \rangle \to e_F - \langle e_{\text{occ}<e_F} \rangle = \frac{2}{5}e_F \quad \text{and} \quad \langle e_p \rangle \to e_i/2 - e_F/5 \]
Marginal hole distributions

The distribution falls off exponentially as expected at forward angles
BUT saturates unexpectedly at back angles.

Exciton model hole distribution tends to a uniform one as the incident energy increases,

\[ p(e_h) \propto e_i - e_h \rightarrow 1/e_F \]

The Fermi gas distribution is also sampled more uniformly as the incident energy increases.
The double differential hole distribution is quite smooth.
The exponential decay is faster for hole states with smaller hole energy, corresponding to larger momentum with respect to the bottom of the well.
The angular distributions saturate at back angles at values larger than expected.

Double differential hole spectrum at $e_i = 100$ MeV (arbitrary units).
Marginal particle distributions

The angular distribution does NOT peak at at 0° and does NOT fall off exponentially, except at higher incident energies and large angles.

Exciton model particle distribution

\[ p(e_p) \propto \begin{cases} 
  e_F & e_p < e_i - e_F \\
  e_i - e_p & e_i - e_F < e_p < e_i 
\end{cases} \]

is very similar to the sampled one.
Double differential particle distribution (Kikuchi-Kawai)

The double differential particle distribution is quite complex. It peaks at a non-zero forward angle for all values of the final energy except the incident one. It is null over a large energy-angular region. It does not decay exponentially with angle.

Double differential particle spectrum at $e_i = 100$ MeV (arbitrary units).
A DDHMS calculation - $p + ^{56}\text{Fe} @ 61\text{ MeV}$

Summary

- We have developed a method to sample the 1p->2p1h and 1h->1p2h transition densities efficiently for MC calculations.
- When used in the DDHMS model, they provide good but not better agreement with data than the usual exciton energy x Chadwick-Oblozinsky angular distributions - at least not yet.
- Work still needs to be done on the effective cross section / mean free path and geometric effects.
- The MC sampling method can be easily extended to relativistic kinematics.