Quantum Phase Transitions and Nuclear Structure

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Program:

> Shape phase transitions in nuclear structure data
> Models describing shape phase transitions in nuclei
> Playing with the models

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Part 1 of 3

Shape (phase) transitions in nuclear structure data

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> Shape phase transitions in nuclear structure data
  Models describing shape phase transitions in nuclei
  Playing with the models
Ground-state QPT signatures in nuclei

$E(4^{+}_1) \text{ [MeV]}$

$E(2^{+}_1) \text{ [MeV]}$

Shape-phase transition!

rotational 3.33

vibrational 2.00

All nuclei with $E(2^{+}_1) < 600 \text{ keV}$

Casten et al., PRL 71, 227 (1993)
Ground-state QPT signatures in nuclei

Semi-empirical criterion for a spherical-to-deformed transition:

\[
P = \frac{N_p N_n}{N_p + N_n} \approx 5
\]

\(N_p, N_n = \text{numbers of valence protons, neutrons or the respective holes}\)

Casten et al., PRL 58, 658 (1987),
McCutchan et al., PRC 69, 024308 (2004)
Ground-state QPT signatures in nuclei

Energy ratio

\[ R_{4/2} = \frac{E(4^+_1)}{E(2^+_1)} \]

Zamfir et al., PRC 66, 021304 (2002)

McCutchan et al., PRC 69, 024308 (2004)
Ground-state QPT signatures in nuclei

$2n$ separation energies

\[ S_{2n} = M(Z, N - 2) + 2m_n c^2 - M(Z, N) \]

Dieperink, Scholten, Iachello, PRL 44, 1747 (1980)
García-Ramos et al., NPA 688, 735 (2001)

Energy ratio

\[ R_{4/2} = \frac{E(4^+_1)}{E(2^+_1)} \]

Zamfir et al., PRC 66, 021304 (2002)

Transition strength

\[ B(E2, 2^+_1 \rightarrow 0^+_1) \]

Iachello, Zamfir, PRL 92, 212501 (2002)
Ground-state QPT signatures in nuclei

$2n$ separation energies

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Ground-state QPT signatures in nuclei

Prolate – oblate transition

<table>
<thead>
<tr>
<th>( N )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleus</td>
<td>(^{200}\text{Hg})</td>
<td>(^{198}\text{Hg})</td>
<td>(^{196}\text{Pt})</td>
<td>(^{194}\text{Pt})</td>
<td>(^{192}\text{Os})</td>
</tr>
<tr>
<td>( \chi )</td>
<td>0.61</td>
<td>0.30</td>
<td>0.10</td>
<td>0.00</td>
<td>−0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( N )</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{190}\text{Os})</td>
<td>(^{188}\text{Os})</td>
<td>(^{186}\text{W})</td>
<td>(^{184}\text{W})</td>
<td>(^{182}\text{W})</td>
<td>(^{180}\text{Hf})</td>
<td></td>
</tr>
<tr>
<td>( \chi )</td>
<td>−0.20</td>
<td>−0.26</td>
<td>−0.30</td>
<td>−0.39</td>
<td>−0.50</td>
<td>−0.60</td>
</tr>
</tbody>
</table>

Part 2 of 3

Models describing shape phase transitions in nuclei

Program:
Shape phase transitions in nuclear structure data
> Models describing shape phase transitions in nuclei
Playing with the models
Geometric (Collective) Model (GCM)

\[ H = \frac{\sqrt{5}}{2K} [\pi \times \pi]^{(0)} + \ldots + \sqrt{5} A [\alpha \times \alpha]^{(0)} - \sqrt{\frac{35}{2}} B [\alpha \times \alpha]^{(2)} \alpha ]^{(0)} + 5C [\alpha \times \alpha]^{(0)} \alpha ]^{(0)} + \ldots \]

\[ V = A \beta^2 + B \beta^3 \cos 3\gamma + C \beta^4 \]

\( \alpha \) ... quadrupole tensor of collective coordinates
2 shape parameters: \( \beta, \gamma \)
3 Euler angles
\( \pi \) ... corresponding tensor of momenta

\( \gamma \)-soft

oblate, prolate, spherical

spinodal critical
Geometric (Collective) Model (GCM)

A. Bohr (1952)
W. Greiner… (1971)

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H = \frac{\sqrt{5}}{2K}[\pi \times \pi]^{(0)} + \ldots + \sqrt{5}A[\alpha \times \alpha]^{(0)} - \sqrt{\frac{35}{2}}B[\alpha \times \alpha]^{(2)} + 5C[\alpha \times \alpha]^{(0)} + \ldots
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Interacting Boson Model (IBM)  

F. Iachello, A. Arima (1975)

\[ H = \sum_{i,j} u^i_j b^+_i b_j + \sum_{i,j,k,l} v^i_{jkl} b^+_i b^+_j b_k b_l = \sum_i w_i C[G_i] \]

Casimir invariants of U(6) subgroups U(5), O(6), SU(3), O(5), O(3)

Phase transitions caused by competing dynamical symmetries

General \( H \) (7 parameters)

Simplified \( H \) (2 parameters)

U(6)...spectrum generating (dynamical) algebra
O(3)...invariant symmetry algebra

Interacting Boson Model (IBM)  

F. Iachello, A. Arima (1975)

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H = \sum_{i,j} u_{ij} b_i^+ b_j + \sum_{i,j,k,l} v_{ijkl} b_i^+ b_j^+ b_k b_l = \sum_i w_i C[G_i]
\]

Phase transitions caused by competing dynamical symmetries:

- \(U(6)\)
- \(U(5)\) \(\rightarrow\) \(O(6)\) \(\rightarrow\) \(SU(3)\)
- \(U(5)\) \(\rightarrow\) \(O(5)\)
- \(O(3)\)

\(U(6)\)...spectrum generating (dynamical) algebra
\(O(3)\)...invariant symmetry algebra

Fermionic Models

- **Phenomenological**
  - Lipkin model  Lipkin, Meshkov, Glick (1965)
  - Fermion Dynamical Symmetry Model  Ginocchio, Wu, Zhang, Guidry… (1980, 86, 87, 88)
  - Pairing models  Chen, Rowe… (1990…), Volya, Zelevinsky (2003), Clark et al. (2006) …

- **Microscopic** *
  - “Collapse of RPA”  Thouless (1960) …
  - Early shell model attempts  Federman, Pittel, Campos (1979) …
  - Monte Carlo shell model  Shimizu, Otsuka, Mizusaki, Honma PRL86, 1171 (2001)
  - Relativistic mean-field calculations  Nikšić, Vretenar, Lalazissis, Ring PRL99, 092502 (2007)

* In the microscopic case the infinite-size limit cannot be performed ⇒ all changes are smoothened by quantum fluctuations.
Part 3 of 3

Playing with the models

Learning new physics on quantum phase transitions

Program:

Shape phase transitions in nuclear structure data

Models describing shape phase transitions in nuclei

> Playing with the models
New types of symmetries at & around the critical point

Critical-point dynamical symmetry
Analytical solutions which are approximately valid at the QPT critical point
First noted by Ginocchio et al. (1987) in the Fermion Dynamical Symmetry Model

Partial dynamical symmetry
At the 1st order critical point: distinct subsets of states retain competing dynamical symmetries
The PDS idea originally introduced in quantum chaos Leviatan et al. (1992……2007)

Dynamical Symmetry I
Phase I
Quasidynamical Symmetry I

Phases I & II
Dynamical Symmetry II

Quasidynamical Symmetry
Extensions of approximate dynamical symmetries far away from the corresponding limits
QDS “is an expression of the possibility that a subset of physical data may exhibit all the properties that would result if the system had a symmetry which, in fact, it does not have.”
Additional degrees of freedom, IBM extensions

- **Proton-neutron variables (IBM-2)**

- **Odd fermions (IBFM)**
  - Jolie et al. (2004)
  - Iachello (2005)
  - Alonso et al. (2005, 2006, 2007)

- **Higher order interactions**
  - Iachello (2004)
  - Thiamova, Cejnar (2006)

- **Other types of bosons**
  - Devi, Kota (1990)
  - Cejnar, Iachello (2007)

- **Configuration mixing**
  - Frank, Van Isacker, Iachello (2006)
  - Hellemans et al. (2007)

- **External rotation**
Finite-size scaling exponents

Calculations beyond the mean field

Example: Energy gap \( \Delta = E_1 - E_0 \)

1\textsuperscript{st} order \( \Delta_c \propto e^{-aN} \)

2\textsuperscript{nd} order \( \Delta_c \propto N^{-1/3} \)

Lipkin Hamiltonian Vidal et al., PRC73,054305(2006)
**Mechanism of the 1st × 2nd order transitions**

Thermodynamic analogy for quantum phase transitions

**Zeros of $Z(T)$ in complex $T$**

<table>
<thead>
<tr>
<th>Partition function</th>
<th>$Z(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free energy</td>
<td>$F(T) = -T \ln Z(T)$</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$C(T) = -T \frac{d^2}{dT^2} F(T)$</td>
</tr>
<tr>
<td>Latent heat</td>
<td>$Q(T) = \lim_{\epsilon \to 0^+} \int_{T-\epsilon}^{T+\epsilon} C(T')dT'$</td>
</tr>
</tbody>
</table>

**Degeneracies of $H(\lambda)$ in complex $\lambda$**

\[
[D_k(\lambda)]^{1/\Omega} = \left[ \prod_{i(k)} [E_i(\lambda) - E_k(\lambda)] \right]^{1/\Omega} \\
U_k(\lambda) = \frac{1}{\Omega} \ln D_k(\lambda) \\
C_k(\lambda) = -\frac{d^2}{d\lambda^2} U_k(\lambda) \\
Q_k(\lambda) = \lim_{\epsilon \to 0^+} \int_{\lambda-\epsilon}^{\lambda+\epsilon} C_k(\lambda')d\lambda'
\]

Yang, Lee (1952) ... Cejnar et al. (2005, 2007)

**Example: linear arrangement of degeneracies (zeros)**

Density near $\text{Im} \lambda \approx 0$

$\rho \propto (\text{Im} \lambda)^\alpha$

- $\alpha > 1$
- $0 < \alpha < 1$
- $\alpha = 0$

<table>
<thead>
<tr>
<th>$Q_k(\lambda_c)$</th>
<th>$\alpha$</th>
<th>1st order</th>
<th>continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_k(\lambda_c) \neq 0$</td>
<td>$\alpha = 0$</td>
<td>$1st$ order</td>
<td>$\text{continuous}$</td>
</tr>
<tr>
<td>$Q_k(\lambda_c) = 0$</td>
<td>$\alpha &gt; 0$</td>
<td>$1st$ order</td>
<td>$\text{continuous}$</td>
</tr>
</tbody>
</table>

$C(\lambda_c) = \infty$, $\alpha \in (0,1]$  
$\frac{d^2}{dx^2} C(\lambda_c) = \infty$, $\alpha \in (1,3]$  
$\frac{d^4}{dx^4} C(\lambda_c) = \infty$, $\alpha \in (3,7]$...
Excited-state quantum phase transitions

Example: 1D Cusp Hamiltonian

$$\hat{H} = -\frac{K^2}{2} \frac{d^2}{dx^2} + x^4 + ax^2 + bx$$

Cejnar, Heinze, Macek, Jolie, Dobeš (2006, 2007)
Caprio, Cejnar, Iachello (2008)
Cejnar, Stránský (2008)
Question: Do quantum shape-phase transitions really exist in nuclei?

Tentative answer: They would exist if nuclei were infinite objects. Finite nuclei only show QPT precursors.

Real QPTs can be studied in various nuclear models ⇒ benefit for both nuclear structure theory & QPT theory (further applications in molecular & mesoscopic physics)

Thanks to collaborators:
P Stránský, M Macek, J Dobeš [Praha]
S Heinze, J Jolie … [Köln]
M Caprio, F Iachello … [Notre Dame, Yale]

Thank you for attention.