Quantum Phase Transitions

and Nuclear Structure

Pavel Cejnar

Institute of Particle & Nuclear Physics, Charles University, Praha, CZ

`cejnar @ ipnp.troja.mff.cuni.cz`

Program:

- > Shape phase transitions in nuclear structure data
- > Models describing shape phase transitions in nuclei
- > Playing with the models

Marie & Pierre Curie 15th Nuclear Physics Workshop, Kazimierz 2008

➢ Part 1 of 3

Shape (phase) transitions in nuclear structure data

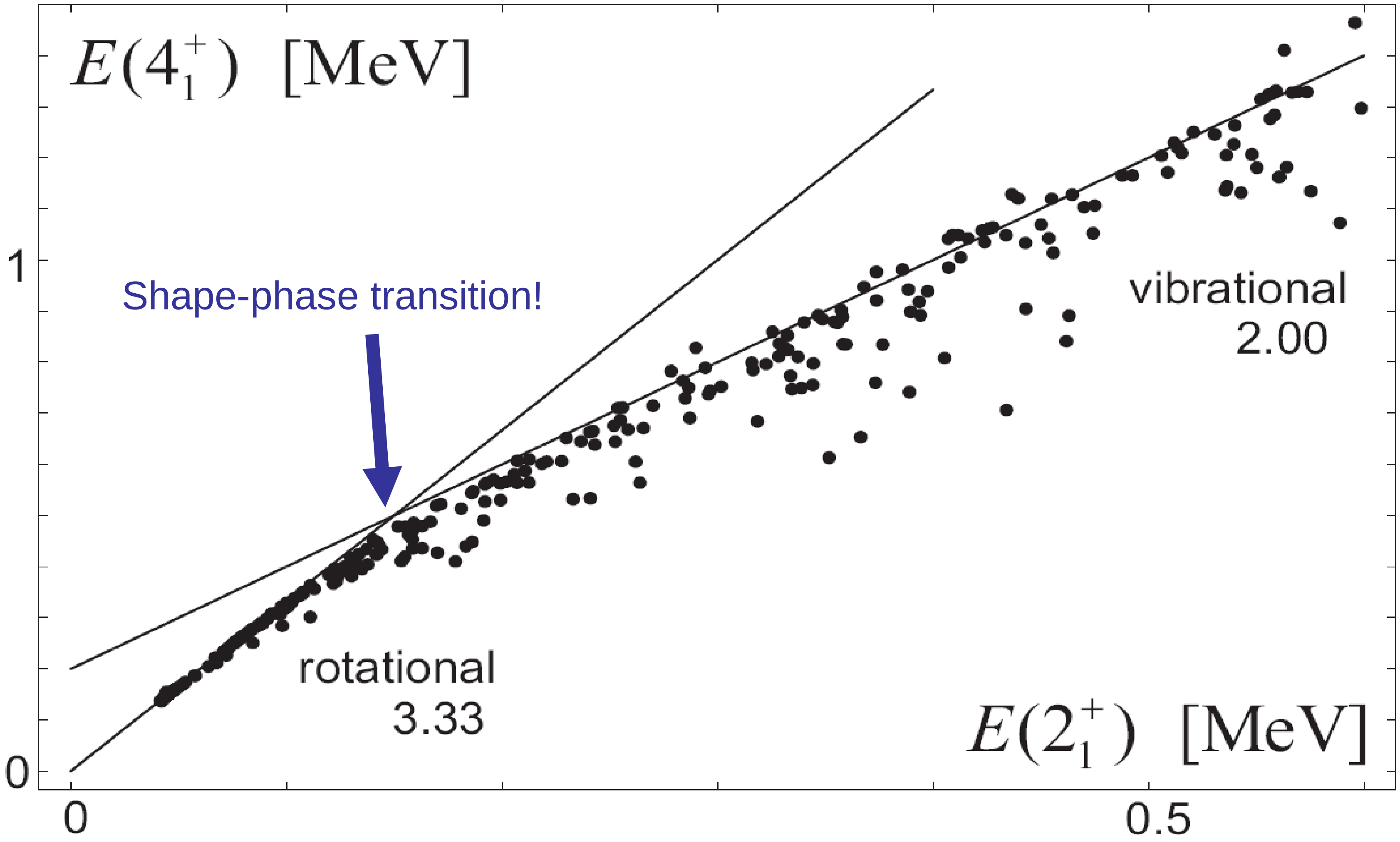
Program:

\> Shape phase transitions in nuclear structure data

Models describing shape phase transitions in nuclei

Playing with the models

Ground-state QPT signatures in nuclei



Shape-phase transition!

All nuclei with $E(2_1^+) < 600\,\mathrm{keV}$

Casten *et al.*, PRL 71,227 (1993)

Ground-state QPT signatures in nuclei

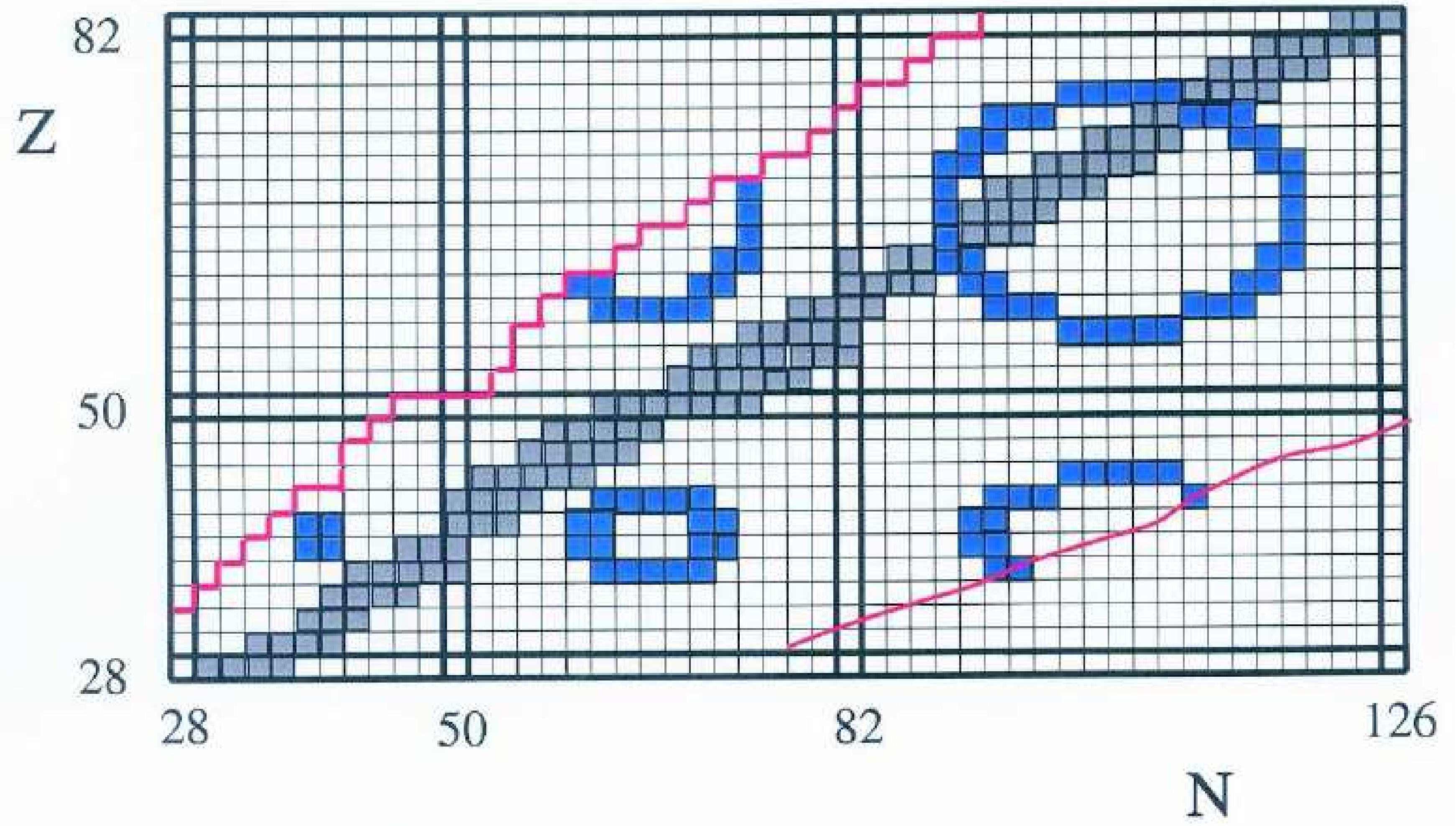
Semi-empirical criterion for a spherical-to-deformed transition:

P-factor

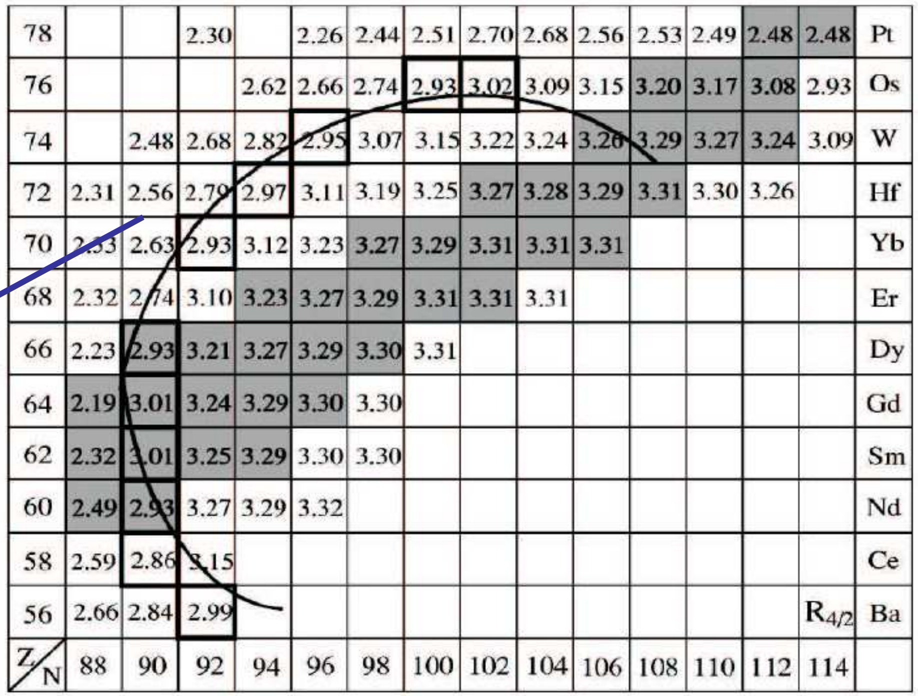
$$P \equiv \frac{N_p N_n}{N_p + N_n} \approx 5$$

Casten *et al.*, PRL 58,658 (1987),
McCutchan *et al.*, PRC 69,024308 (2004)

(N_p, N_n = numbers of valence protons, neutrons or the respective holes)



Ground-state QPT signatures in nuclei



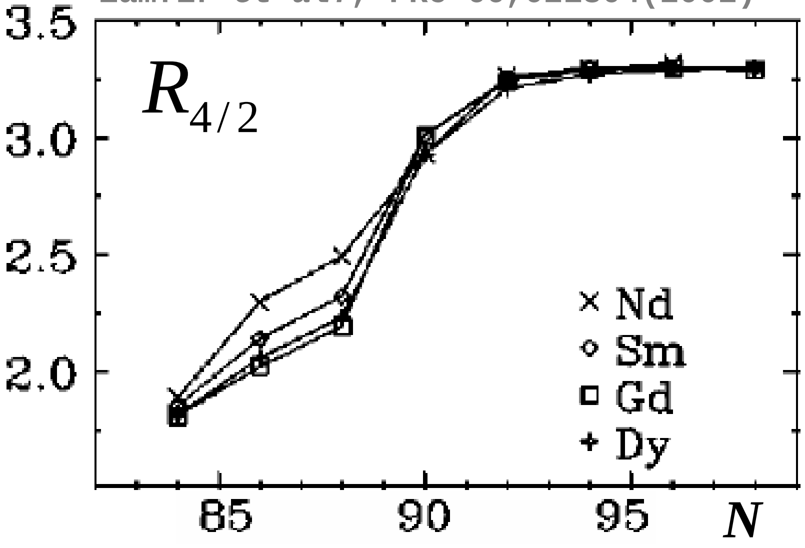
78			2.30		2.26	2.44	2.51	2.70	2.68	2.56	2.53	2.49	2.48	2.48	Pt
76				2.62	2.66	2.74	2.93	3.02	3.09	3.15	3.20	3.17	3.08	2.93	Os
74		2.48	2.68	2.82	2.95	3.07	3.15	3.22	3.24	3.26	3.29	3.27	3.24	3.09	W
72	2.31	2.56	2.79	2.97	3.11	3.19	3.25	3.27	3.28	3.29	3.31	3.30	3.26		Hf
70	2.33	2.63	2.93	3.12	3.23	3.27	3.29	3.31	3.31	3.31					Yb
68	2.32	2.74	3.10	3.23	3.27	3.29	3.31	3.31	3.31						Er
66	2.23	2.93	3.21	3.27	3.29	3.30	3.31								Dy
64	2.19	3.01	3.24	3.29	3.30	3.30									Gd
62	2.32	3.01	3.25	3.29	3.30	3.30									Sm
60	2.49	2.93	3.27	3.29	3.32										Nd
58	2.59	2.86	3.15												Ce
56	2.66	2.84	2.99											$R_{4/2}$	Ba
Z/N	88	90	92	94	96	98	100	102	104	106	108	110	112	114	

McCutchan *et al.*, PRC 69,024308 (2004)

Energy ratio

$$R_{4/2}=\frac{E(4_1^+)}{E(2_1^+)}$$

Zamfir *et al.*, PRC 66,021304(2002)

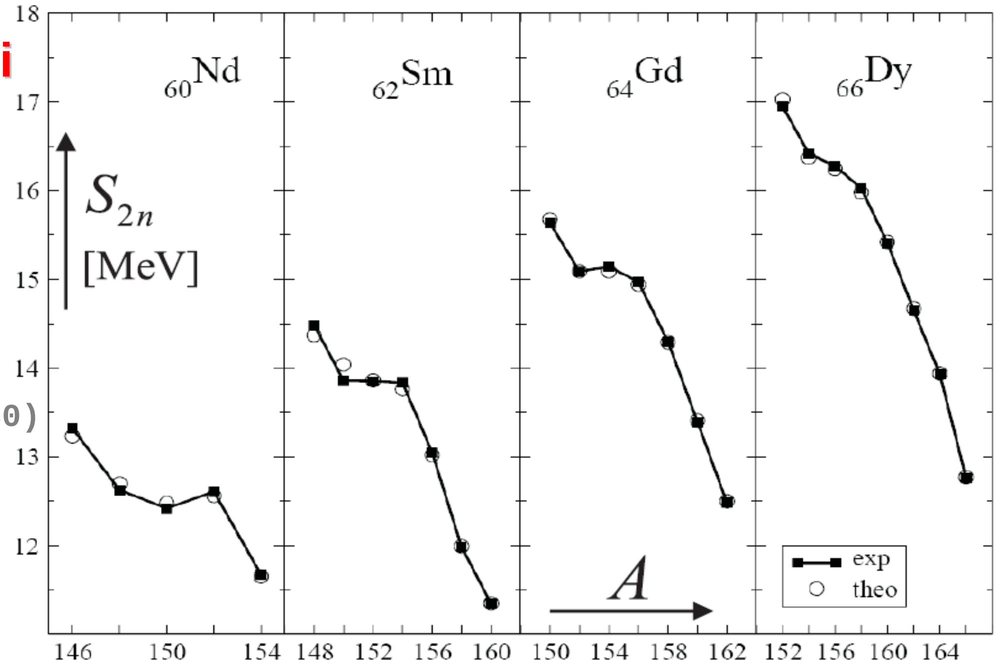


Ground-state QPT signatures in nuclei

***2n* separation energies**

$$S_{2n} = M(Z,N-2) + 2m_n c^2 - M(Z,N)$$

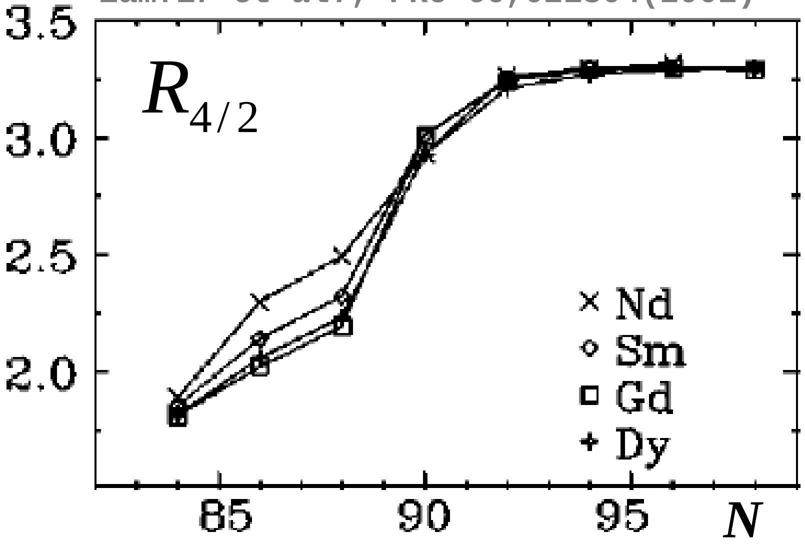
Dieperink,Scholten,Iachello, PRL 44,1747(1980)
García-Ramos *et al.*, NPA 688,735(2001)
García-Ramos *et al.*, PRC 68,024307(2003)



Energy ratio

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

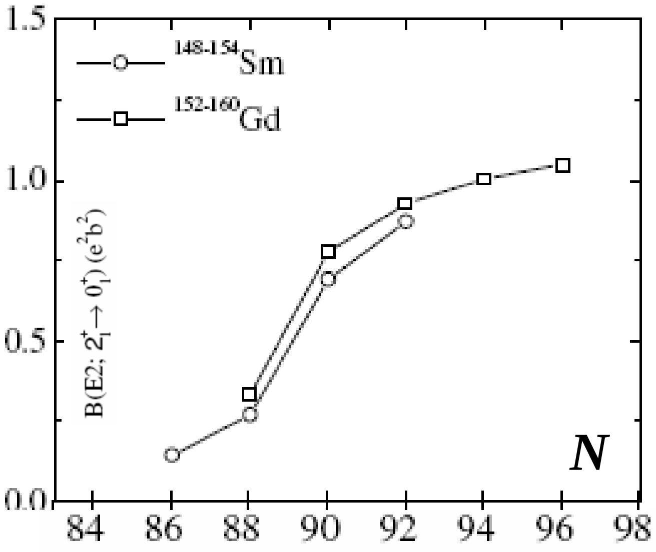
Zamfir *et al.*, PRC 66,021304(2002)



Transition strength

$$B(E2, 2_1^+ \rightarrow 0_1^+)$$

Iachello,Zamfir,PRL92,212501(2002)

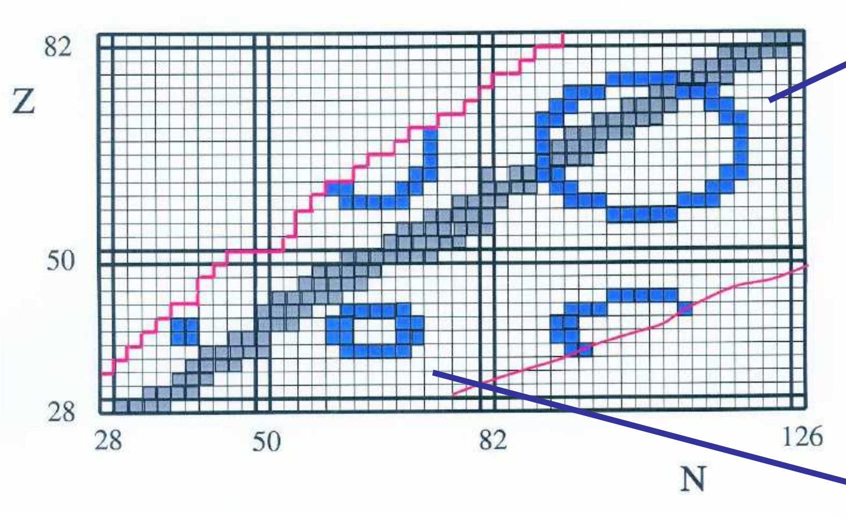


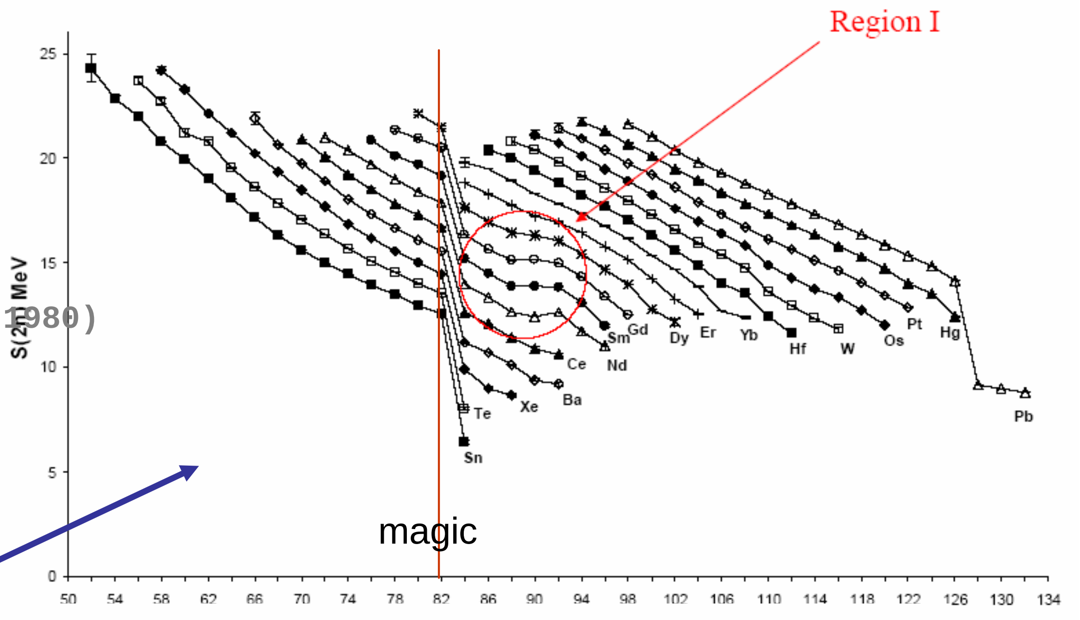
Ground-state QPT signatures in nuclei

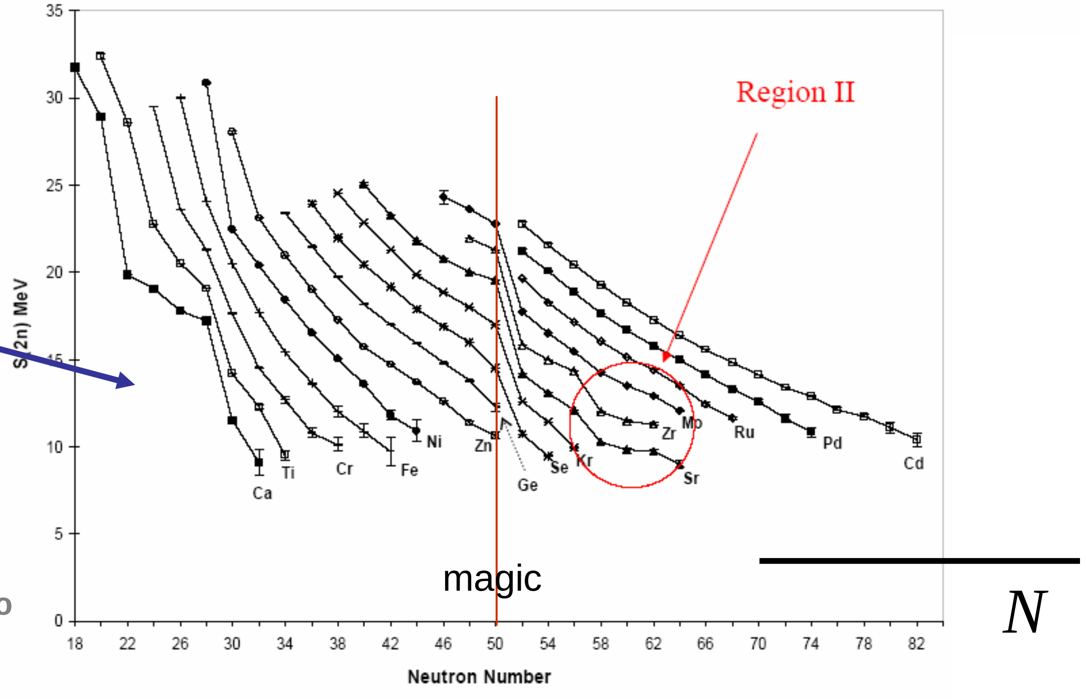
***2n* separation energies**

$$S_{2n} = M(Z,N-2) + 2m_n c^2 - M(Z,N)$$

Dieperink,Scholten,Iachello, PRL 44,1747(1980)
García-Ramos *et al.*, NPA 688,735(2001)
García-Ramos *et al.*, PRC 68,024307(2003)







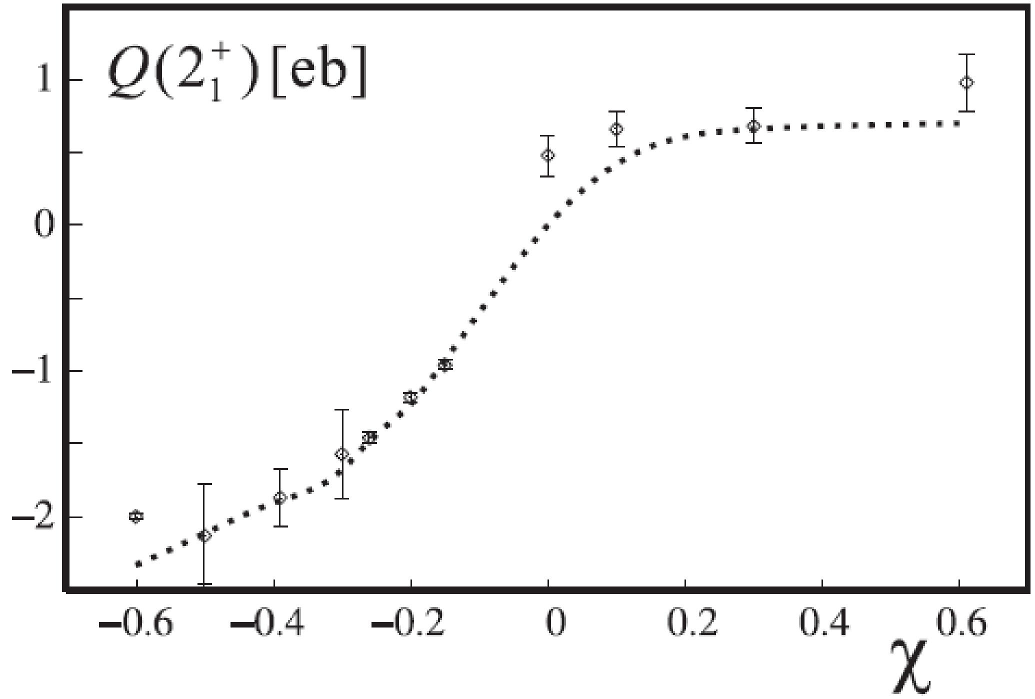
Courtesy: F.Iachello

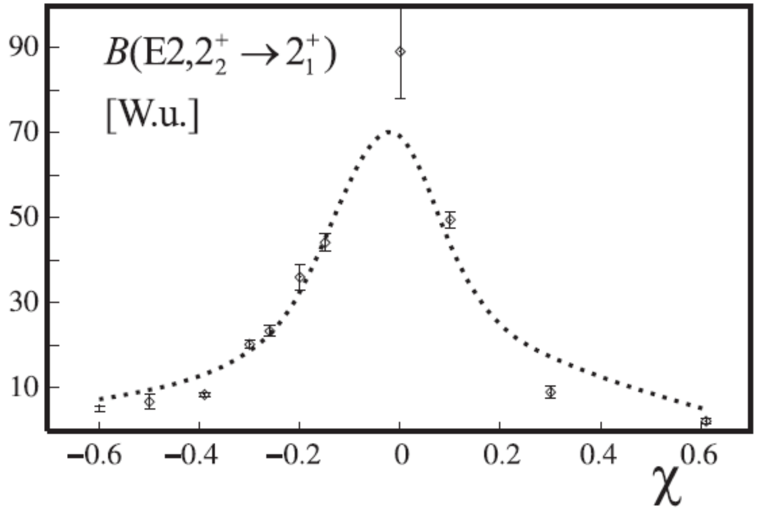
Ground-state QPT signatures in nuclei

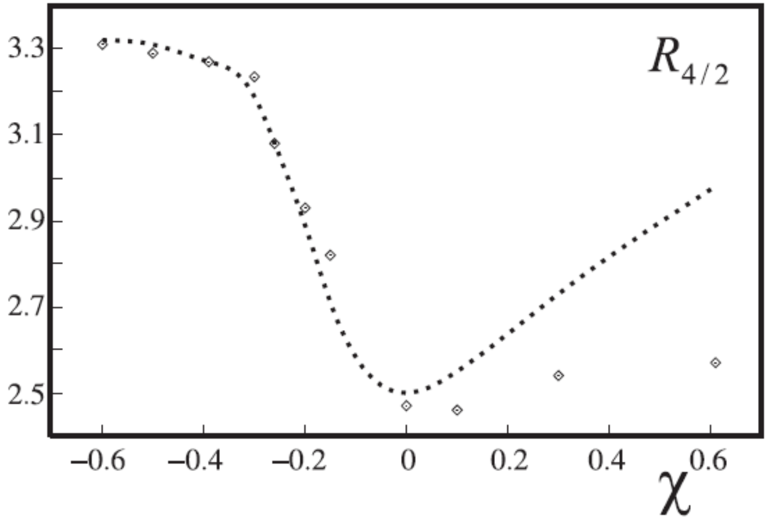
Prolate – oblate transition

N	4	5	6	7	8	9	10	11	12	13	14
Nucleus	^{200}Hg	^{198}Hg	^{196}Pt	^{194}Pt	^{192}Os	^{190}Os	^{188}Os	^{186}W	^{184}W	^{182}W	^{180}Hf
χ	0.61	0.30	0.10	0.00	−0.15	−0.20	−0.26	−0.30	−0.39	−0.50	−0.60

Jolie, Linnemann, PRC 68,031301 (2003)







- Part 2 of 3

Models describing shape phase transitions in nuclei

Program:

Shape phase transitions in nuclear structure data

\> Models describing shape phase transitions in nuclei

Playing with the models

Geometric (Collective) Model (GCM)

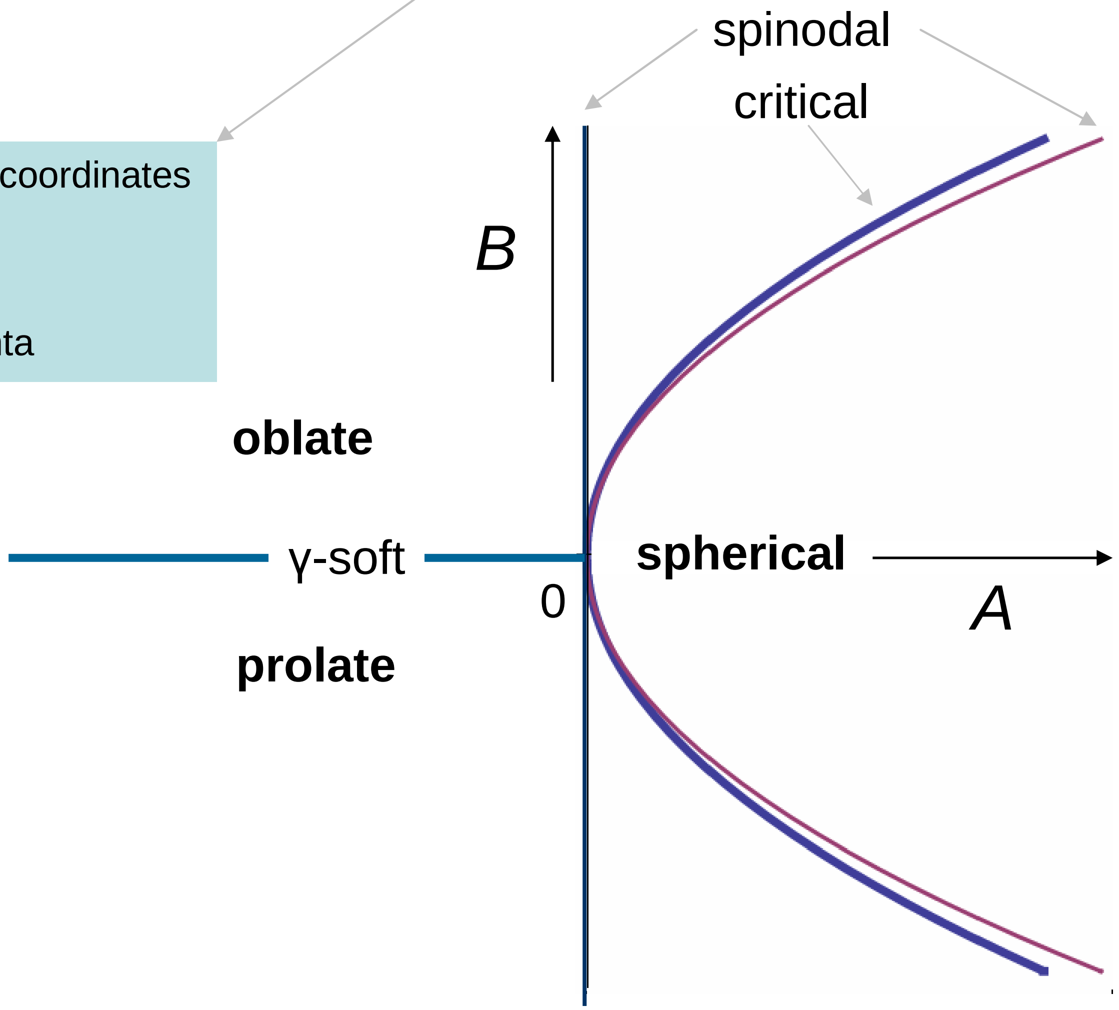
A.Bohr (1952)
W.Greiner… (1971)

$$H = \frac{\sqrt{5}}{2K}[\pi\times\pi]^{(0)} + \ldots\ldots + \sqrt{5}A[\alpha\times\alpha]^{(0)} - \sqrt{\frac{35}{2}}B\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)} + 5C\left([\alpha\times\alpha]^{(0)}\right)^{2} + \ldots.$$

$$V = A\beta^{2} + B\beta^{3}\cos 3\gamma + C\beta^{4}$$

α … quadrupole tensor of collective coordinates
  2 shape parameters: ***β, γ***
  3 Euler angles

π ... corresponding tensor of momenta

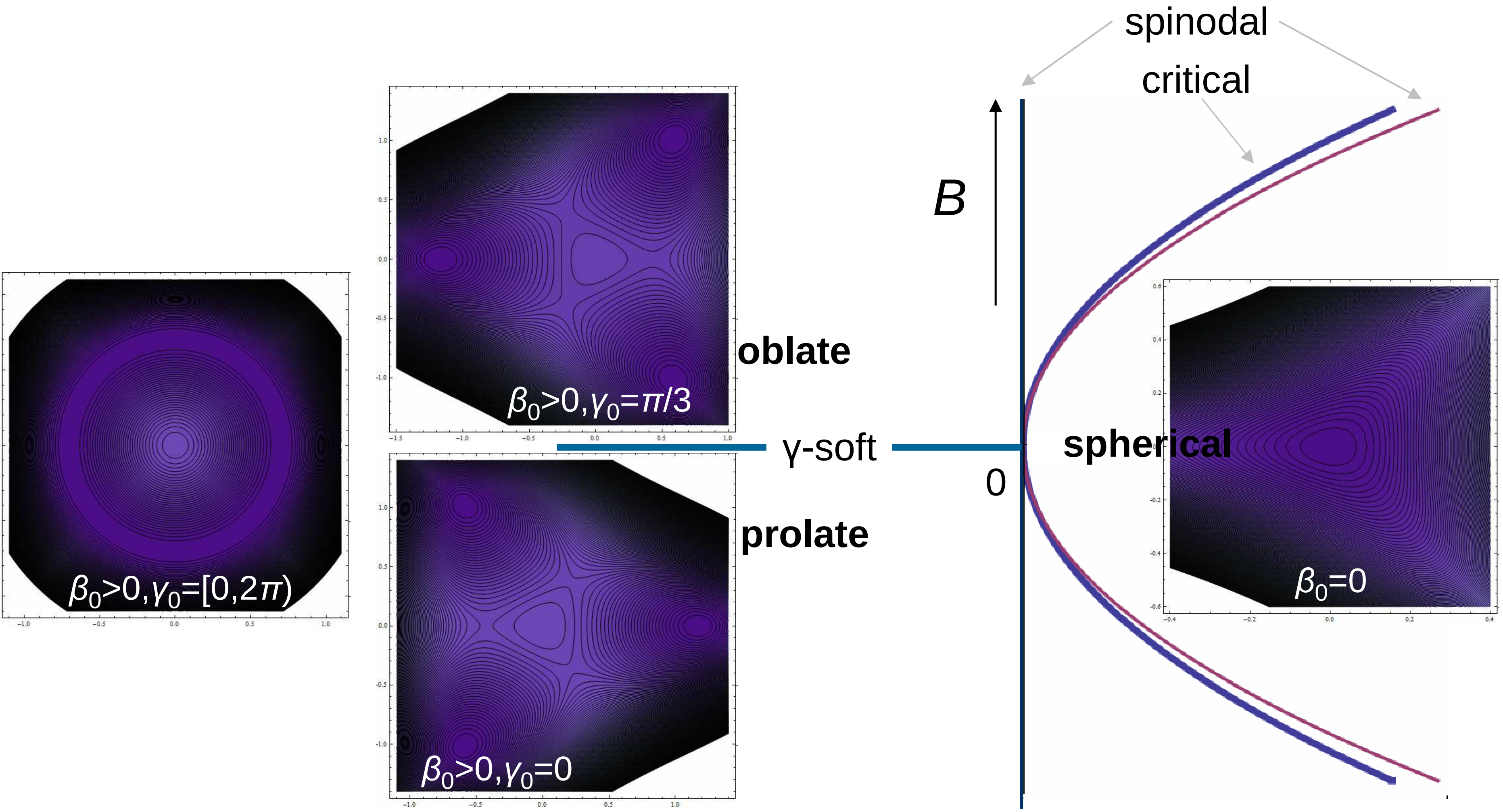


Geometric (Collective) Model (GCM)

A.Bohr (1952)
W.Greiner… (1971)

$$H = \frac{\sqrt{5}}{2K}[\pi\times\pi]^{(0)} + \ldots\ldots + \sqrt{5}A[\alpha\times\alpha]^{(0)} - \sqrt{\frac{35}{2}}B\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)} + 5C\left([\alpha\times\alpha]^{(0)}\right)^2 + \ldots$$

$$V = A\beta^2 + B\beta^3\cos 3\gamma + C\beta^4$$



spinodal

critical

B

oblate

$\beta_0>0,\gamma_0=\pi/3$

γ-soft

$\beta_0>0,\gamma_0=[0,2\pi)$

0

spherical

$\beta_0=0$

prolate

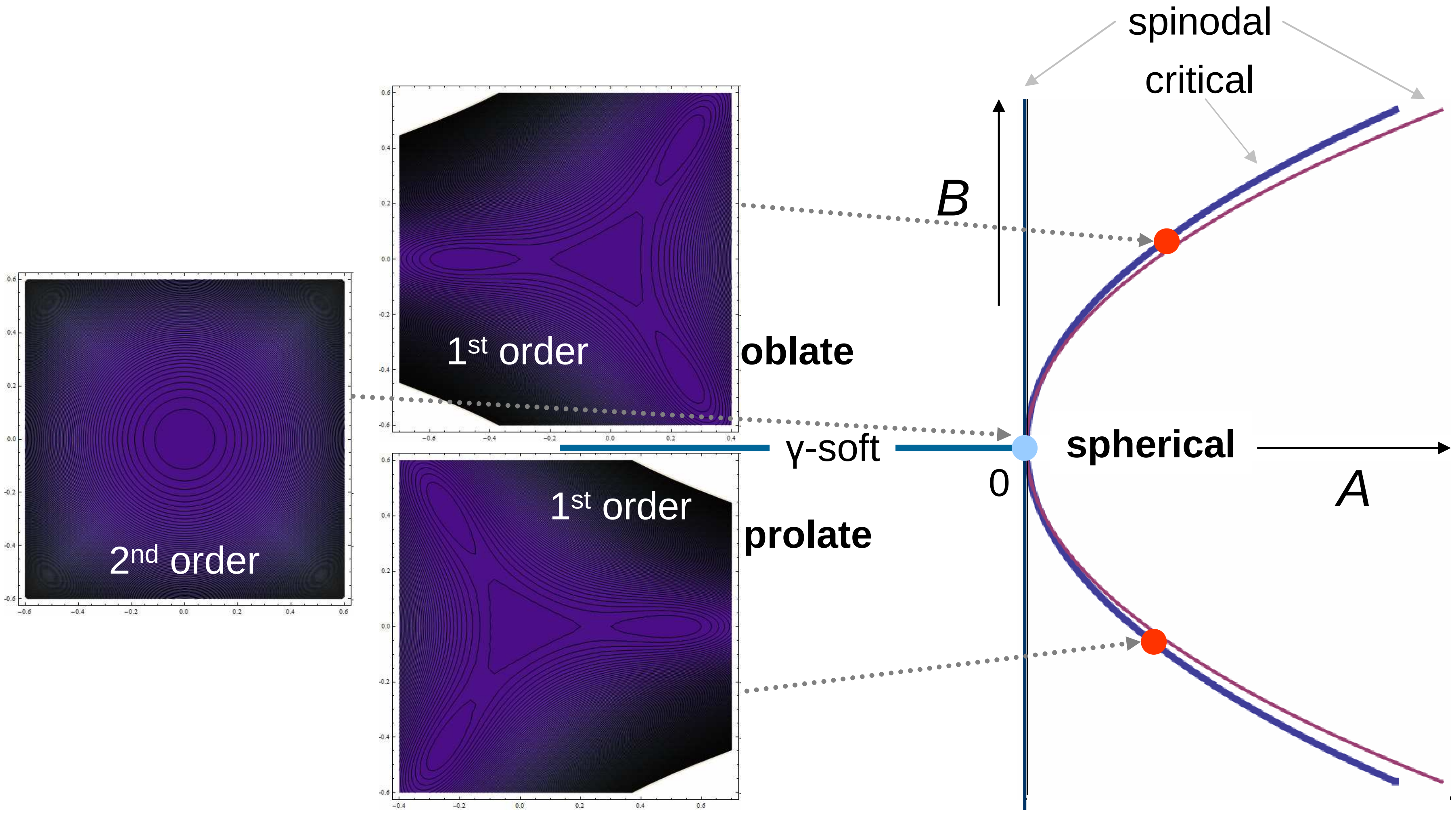
$\beta_0>0,\gamma_0=0$

Geometric (Collective) Model (GCM)

A.Bohr (1952)
W.Greiner… (1971)

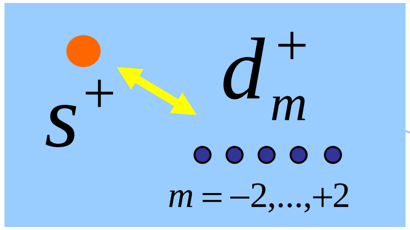
$$H = \frac{\sqrt{5}}{2K}[\pi\times\pi]^{(0)} + \ldots\ldots + \sqrt{5}A[\alpha\times\alpha]^{(0)} - \sqrt{\frac{35}{2}}B\left[[\alpha\times\alpha]^{(2)}\times\alpha\right]^{(0)} + 5C\left([\alpha\times\alpha]^{(0)}\right)^2 + \ldots\ldots$$

$$V = A\beta^2 + B\beta^3\cos 3\gamma + C\beta^4$$



Interacting Boson Model (IBM)

F.Iachello, A.Arima (1975)



$$H=\sum_{i,j} u_{ij} b_i^+ b_j + \sum_{i,j,k,l} v_{ijkl} b_i^+ b_j^+ b_k b_l = \sum_i w_i C[G_i]$$

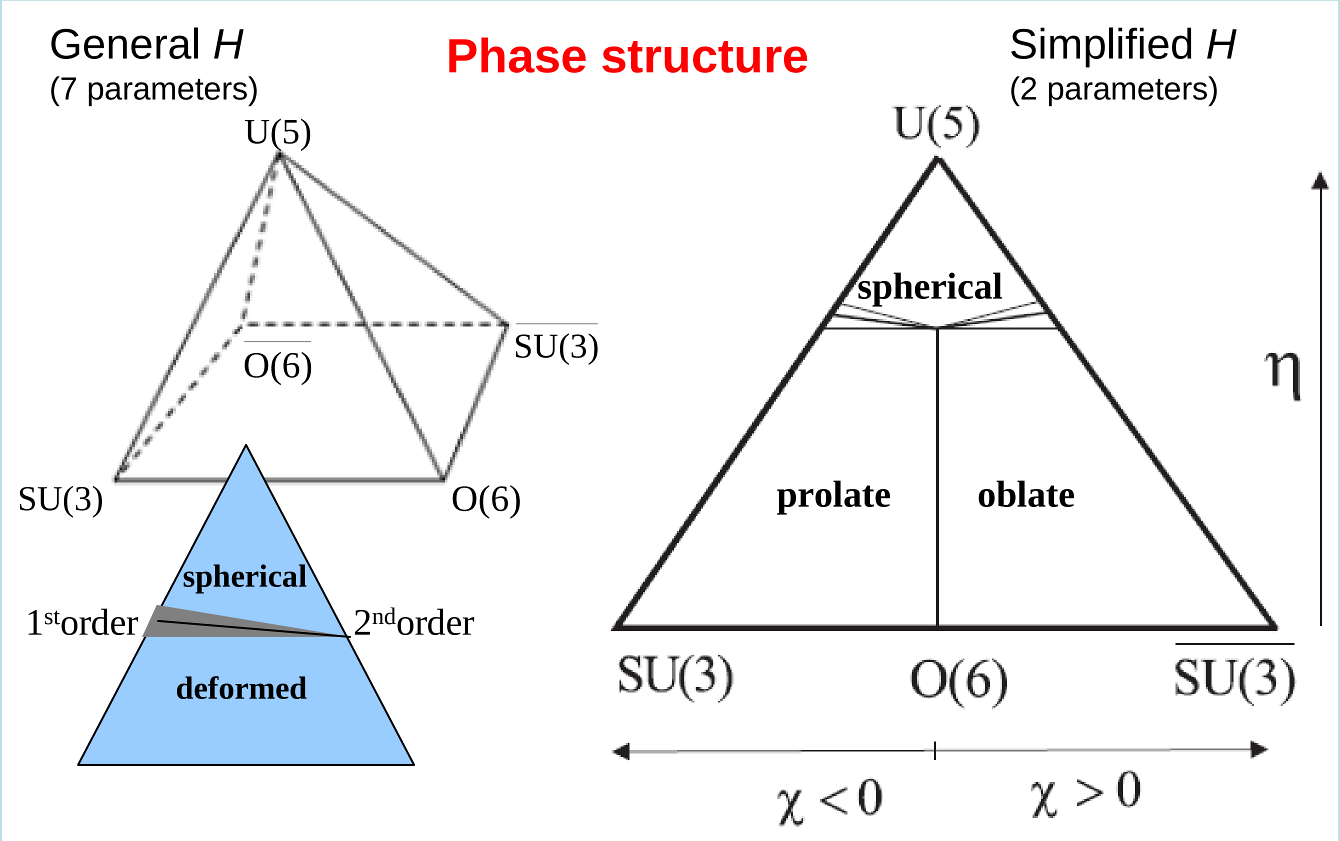
Casimir invariants of U(6) subgroups
U(5),O(6),SU(3),O(5),O(3)

Phase transitions caused by competing **dynamical symmetries**

$$\begin{tikzcd} & U(6) \arrow[dl] \arrow[d] \arrow[dr] & \\ U(5) \arrow[dr] & O(6) \arrow[d] & SU(3) \arrow[ddl] \\ & O(5) \arrow[d] & \\ & O(3) & \end{tikzcd}$$

U(6)…spectrum generating (dynamical) algebra

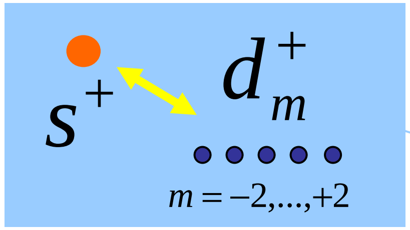
O(3)…invariant symmetry algebra



P.Cejnar,J.Jolie, Prog.Part.Nucl.Phys.(2008); arXiv:0807.3467[nucl-th]

Interacting Boson Model (IBM)

F.Iachello, A.Arima (1975)



$$H=\sum_{i,j} u_{ij} b_i^+ b_j + \sum_{i,j,k,l} v_{ijkl} b_i^+ b_j^+ b_k b_l = \sum_i w_i C[G_i]$$

Casimir invariants of U(6) subgroups
U(5),O(6),SU(3),O(5),O(3)

Phase transitions caused by competing **dynamical symmetries**

$$\begin{tikzcd}
 & U(6) \arrow[dl] \arrow[d] \arrow[dr] & \\
U(5) \arrow[dr] & O(6) \arrow[d] & SU(3) \arrow[ddl] \\
 & O(5) \arrow[d] & \\
 & O(3) &
\end{tikzcd}$$

U(6)…spectrum generating (dynamical) algebra

O(3)…invariant symmetry algebra

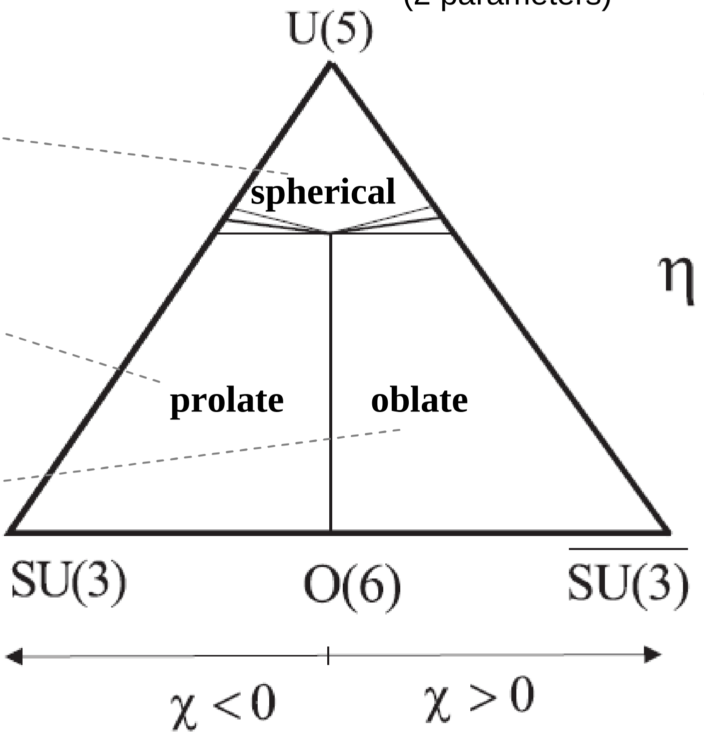
Phase structure

Simplified *H*
(2 parameters)

$$|\Psi_0\rangle \propto (s^+)^N |0\rangle$$

$$|\Psi_0\rangle \propto (s^+ + \beta_0 d_0^+)^N |0\rangle$$

$$|\Psi_0\rangle \propto (s^+ - \beta_0 d_0^+)^N |0\rangle$$



P.Cejnar,J.Jolie, Prog.Part.Nucl.Phys.(2008); arXiv:0807.3467[nucl-th]

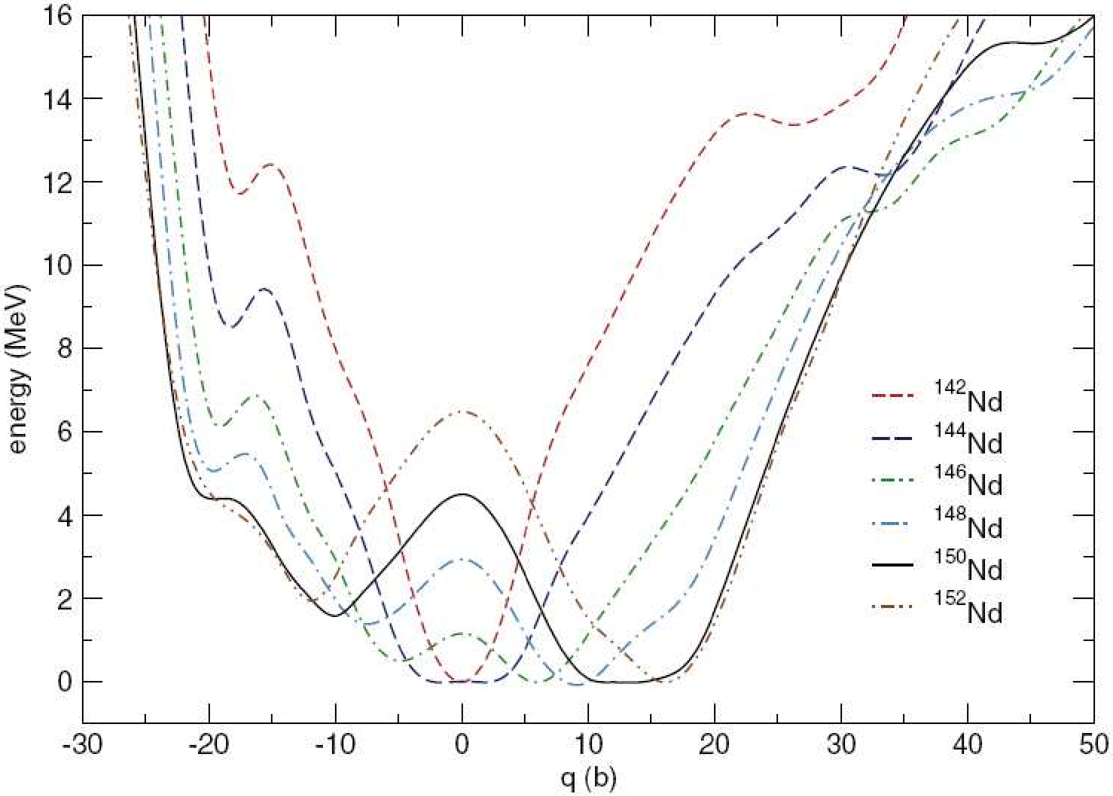
Fermionic Models

➢ Phenomenological

Lipkin model Lipkin, Meshkov, Glick(1965)
Fermion Dynamical Symmetry Model Ginocchio, Wu, Zhang, Guidry…(1980,86,87,88)
Pairing models Chen, Rowe…(1990…), Volya, Zelevinsky(2003), Clark *et al.*(2006)……

➢ Microscopic *

"Collapse of RPA" Thouless(1960)…
Early shell model attempts Federman, Pittel, Campos(1979)…
Monte Carlo shell model Shimizu, Otsuka, Mizusaki, Honma PRL86,1171(2001)
Relativistic mean-field calculations Nikšić, Vretenar, Lalazissis, Ring PRL99,092502(2007)



---

* In the microscopic case the infinite-size limit cannot be performed
⇒ all changes are smoothened by quantum fluctuations.

- **Part 3 of 3**

Playing with the models

Learning new physics on quantum phase transitions

Program:

Shape phase transitions in nuclear structure data

Models describing shape phase transitions in nuclei

\> Playing with the models

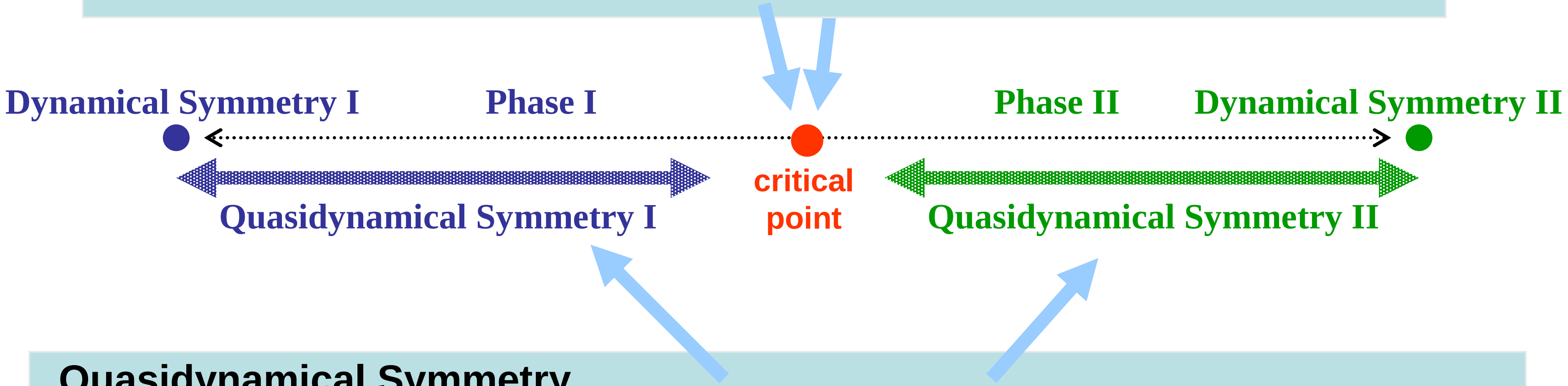
New types of symmetries at & around the critical point

Critical-point dynamical symmetry

Analytical solutions which are approximately valid at the QPT critical point
First noted by Ginocchio *et al.*(1987) in the Fermion Dynamical Symmetry Model
Cast in the geometric framework by Iachello(2000,2001),Bonatsos *et al.*(2004) E(5), X(5), Z(5)…

Partial dynamical symmetry

At the 1st order critical point: distinct subsets of states retain competing dynamical symmetries
The PDS idea originally introduced in quantum chaos Leviatan *et al.*(1992……2007)



Dynamical Symmetry I **Phase I** **Phase II** **Dynamical Symmetry II**

critical point

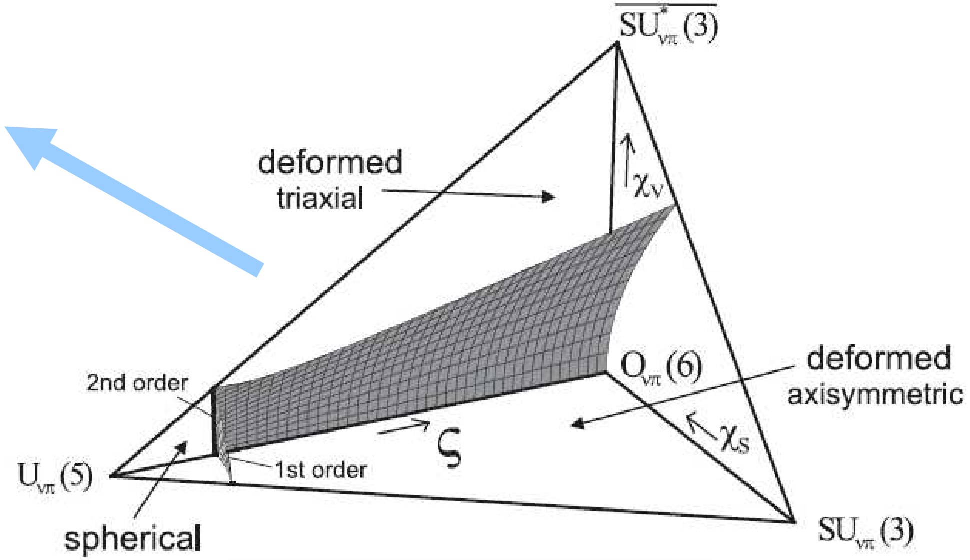
Quasidynamical Symmetry I **Quasidynamical Symmetry II**

Quasidynamical Symmetry

Extensions of approximate dynamical symmetries far away from the corresponding limits
QDS "is an expression of the possibility that a subset of physical data may exhibit all the properties that would result if the system *had* a symmetry which, in fact, it *does not have*."
Rowe *et al.*(1998,2004,2005)

Additional degrees of freedom, IBM extensions

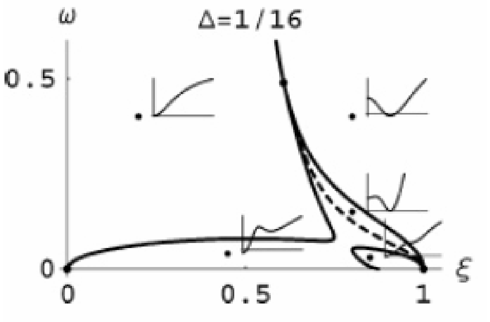
- **Proton-neutron variables** (IBM-2)
  Caprio,Iachello (2004,2005)
  Arias,García-Ramos,Dukelsky (2004)



- **Odd fermions** (IBFM) ?
  Jolie *et al.* (2004)
  Iachello (2005)
  Alonso *et al.* (2005,2006,2007)

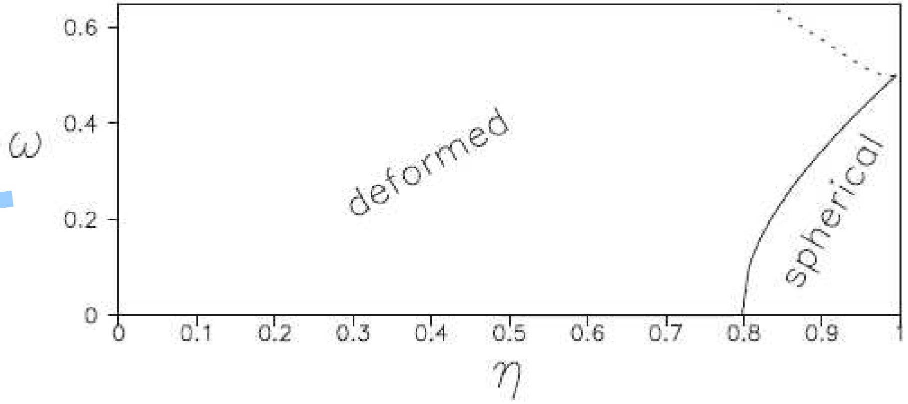
- **Higher order interactions** ?
  Iachello (2004)
  Jolos (2004)
  Thiamova,Cejnar (2006)

- **Other types of bosons** ?
  Devi,Kota (1990)
  Cejnar,Iachello (2007)



- **Configuration mixing**
  Frank,Van Isacker,Iachello (2006)
  Hellemans *et al.* (2007)

- **External rotation**
  Cejnar (2002,2003)



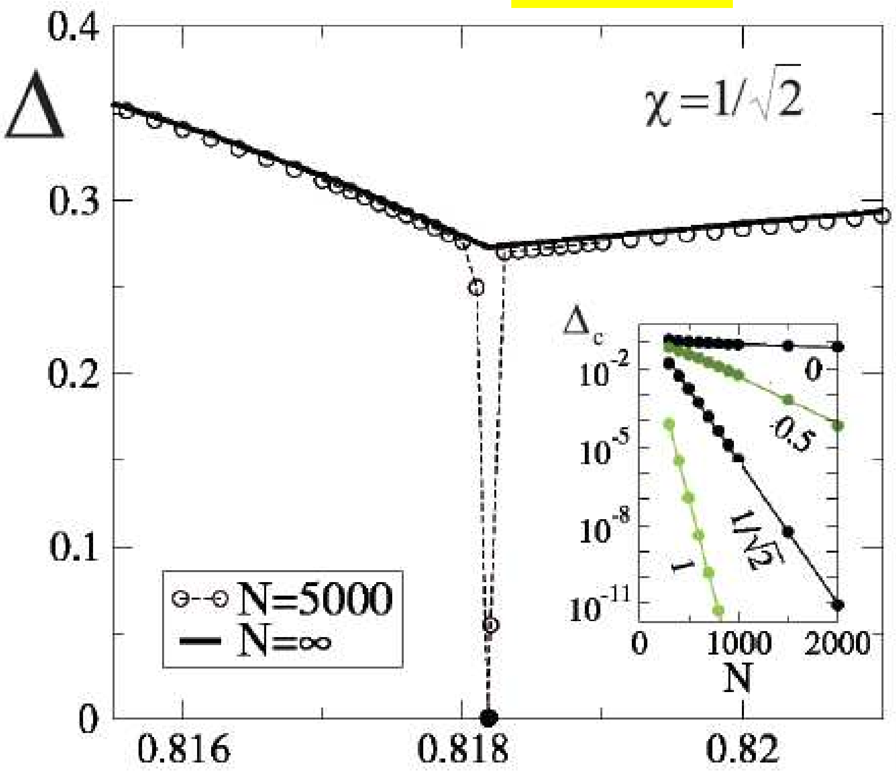
Finite-size scaling exponents

Calculations beyond the mean field

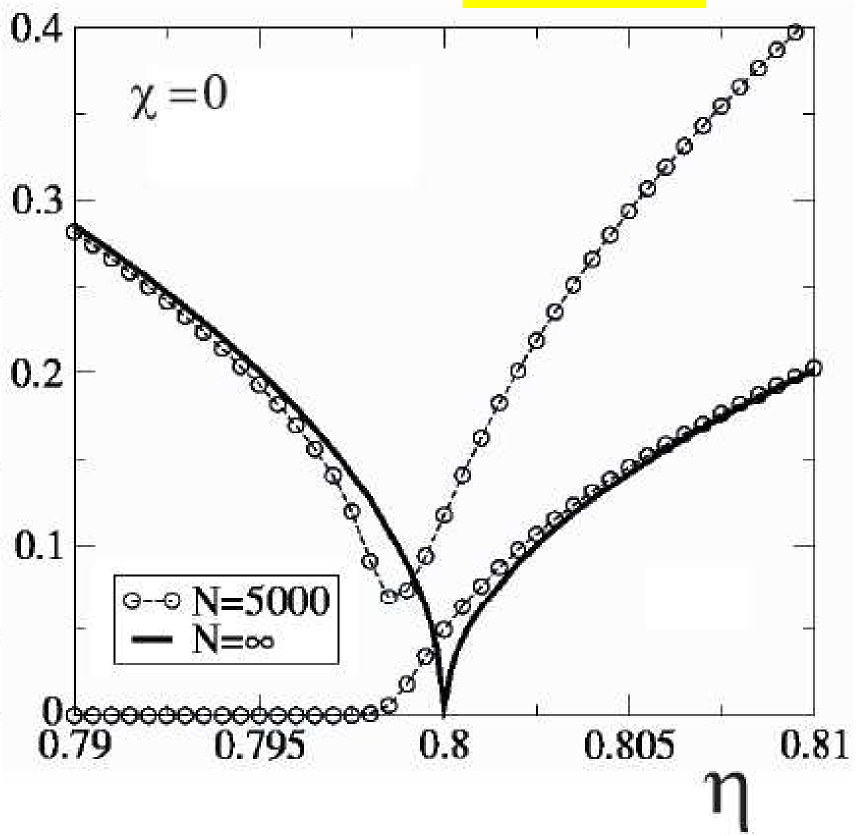
Dusuel, Vidal, Arias, Dukelsky, García-Ramos (2005,2006,2007)

Example: **Energy gap** $\Delta = E_1 - E_0$

1st order $\Delta_c \propto e^{-aN}$



2nd order $\Delta_c \propto N^{-1/3}$



Lipkin Hamiltonian Vidal *et al.*, PRC73,054305(2006)

Mechanism of the 1st x 2nd order transitions

Thermodynamic analogy for quantum phase transitions

Zeros of *Z(T)* in complex *T*

partition function $Z(T)$

free energy $F(T) = -T \ln Z(T)$

specific heat $C(T) = -T \frac{\partial^2}{\partial T^2} F(T)$

latent heat $Q(T) = \lim_{\varepsilon \to 0+} \int_{T-\varepsilon}^{T+\varepsilon} C(T')dT'$

⬌

Degeneracies of *H(λ)* in complex *λ*

$$[D_k(\lambda)]^{1/\Omega} = \left(\prod_{i(\neq k)} [E_i(\lambda) - E_k(\lambda)] \right)^{1/\Omega}$$

$$U_k(\lambda) = \tfrac{-1}{\Omega} \ln D_k(\lambda) \qquad \Omega = n-1$$

$$C_k(\lambda) = -\frac{d^2}{d\lambda^2} U_k(\lambda)$$

$$Q_k(\lambda) = \lim_{\varepsilon \to 0+} \int_{\lambda-\varepsilon}^{\lambda+\varepsilon} C_k(\lambda')d\lambda'$$

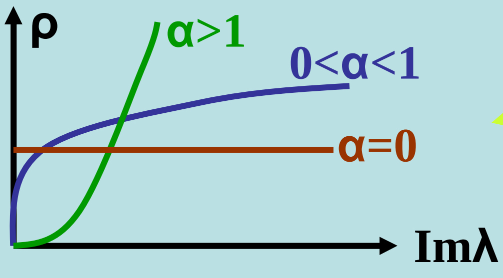
`Yang, Lee (1952)…`

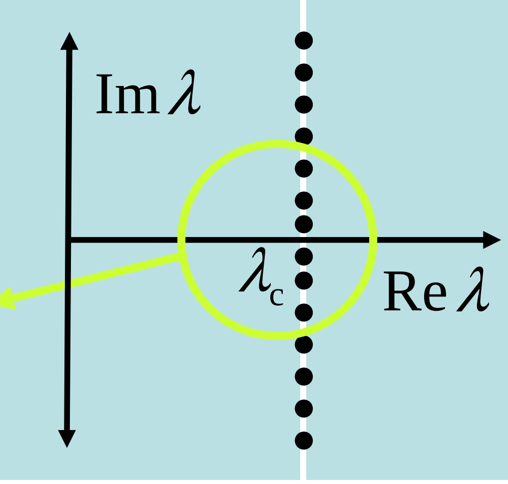
`Cejnar et al. (2005,2007)`

Example: **linear arrangement of degeneracies (zeros)**

density near Imλ≈0

$$\rho \propto (\mathrm{Im}\,\lambda)^\alpha$$





$Q_k(\lambda_c) \neq 0 \quad \alpha = 0$ **1st order**

$Q_k(\lambda_c) = 0 \quad \alpha > 0$ **continuous**

$$C(\lambda_c) = \infty \quad \alpha \in (0,1]$$

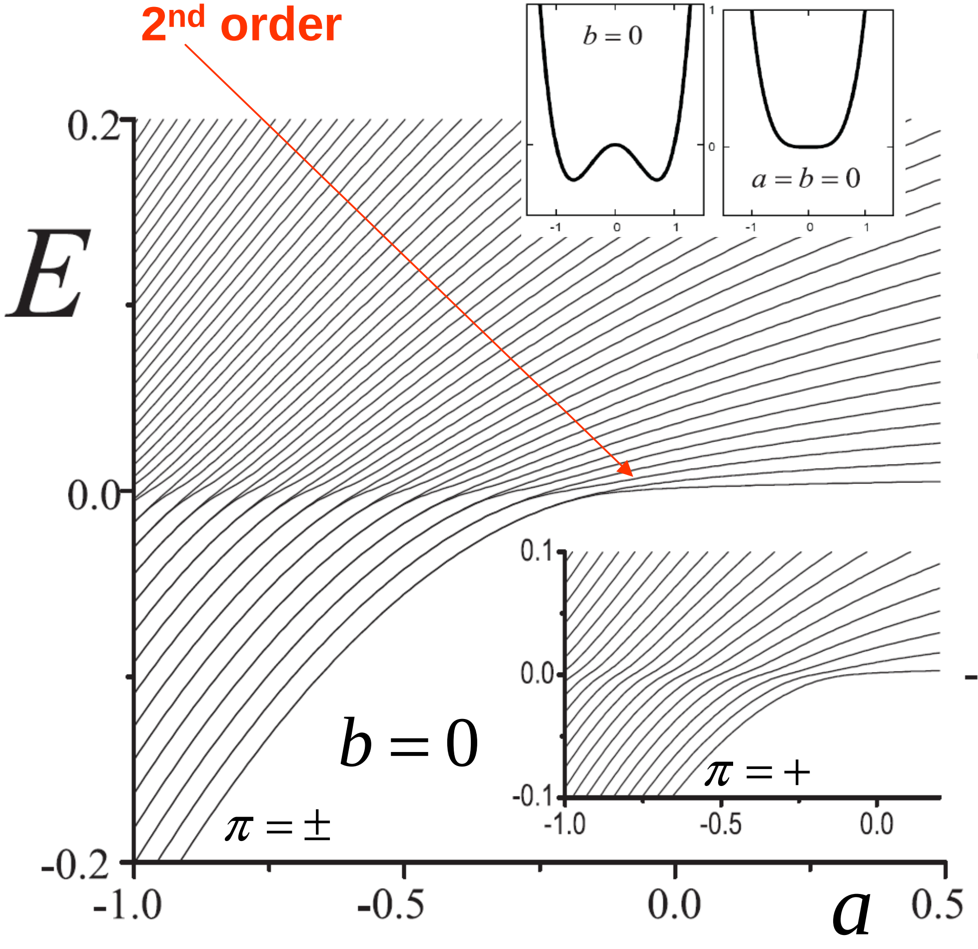
$$\frac{d^2}{dx^2} C(\lambda_c) = \infty \quad \alpha \in (1,3]$$

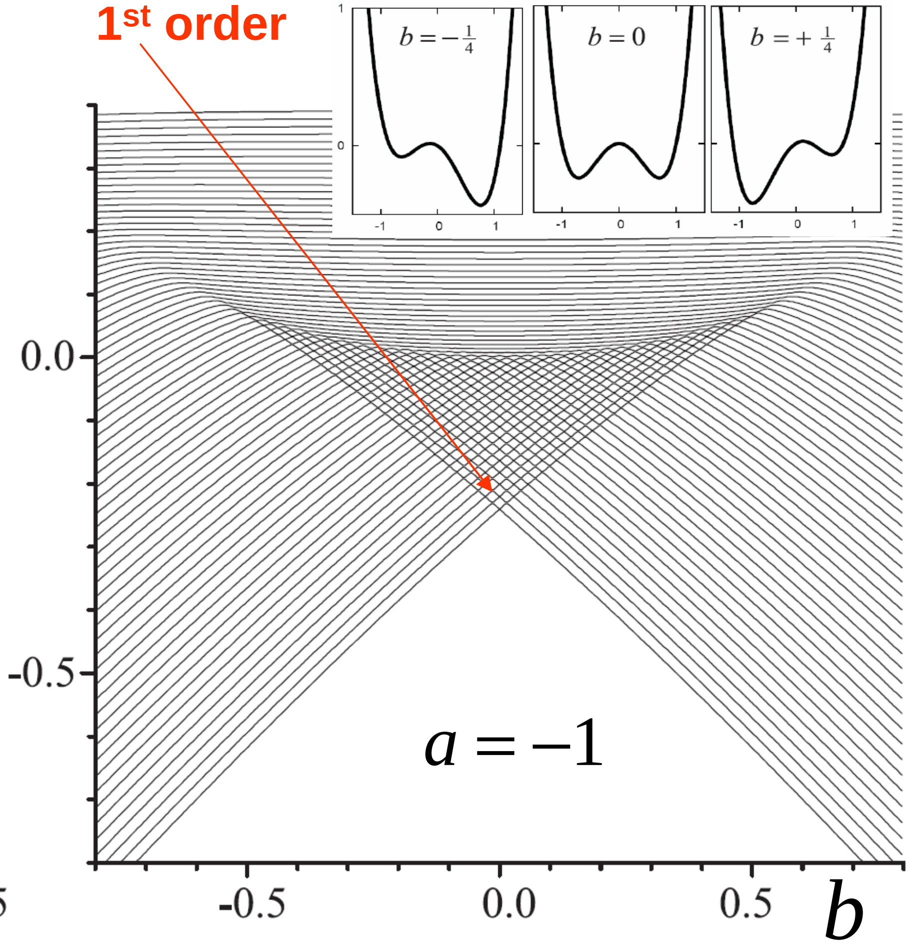
$$\frac{d^4}{dx^4} C(\lambda_c) = \infty \quad \alpha \in (3,7]....$$

Excited-state quantum phase transitions

Cejnar, Heinze, Macek, Jolie, Dobeš (2006,2007)
Caprio, Cejnar, Iachello (2008)
Cejnar, Stránský (2008)

Example: **1D Cusp Hamiltonian** $\hat{H} = -\frac{K^2}{2}\frac{d^2}{dx^2} + x^4 + ax^2 + bx$


2nd order


1st order

Conclusions

Question: **Do quantum shape-phase transitions really exist in nuclei ?**
Tentative answer: They would exist if nuclei were infinite objects.
Finite nuclei only show QPT precursors.

Real QPTs can be studied in various nuclear models
⇒ benefit for both **nuclear structure theory & QPT theory**
(further applications in molecular & mesoscopic physics)

Thanks to collaborators:
P Stránský, M Macek, J Dobeš [Praha]
S Heinze, J Jolie … [Köln]
M Caprio, F Iachello … [Notre Dame, Yale]

Thank you for attention.